

1 Fibonacci

For the following two problems, recall that the Fibonacci sequence is $1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144 \dots$.

Problem 1. By successive differences I mean take two numbers in the sequence that are side by side and subtract the larger from the smaller. For the Fibonacci sequence, find the first 9 successive differences; i.e., start off with $1 - 1$, then $2 - 1$, then $3 - 2$, and so on. If you were to construct a new sequence by taking these successive differences for all of the Fibonacci sequence, how do you think this new sequence would relate to the original Fibonacci sequence?

Extra: If you are comfortable with the following notation, you may use it. Let F_n be the n th Fibonacci number; e.g., $F_1 = 1, F_2 = 1, F_3 = 2, F_4 = 3 \dots$. Then I want you to find $F_{n+1} - F_n$ for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9$.

Solution 1. $1 - 1 = 0, 2 - 1 = 1, 3 - 2 = 1, 5 - 3 = 2, 8 - 3 = 5$, and so on. We see that this new sequence is just (basically) returning the Fibonacci sequence; i.e., taking successive differences of the Fibonacci sequence gives us back another Fibonacci sequence.

Problem 2. Compute $\frac{1+\sqrt{5}}{2}$ with a calculator. You can simply plug in “(1+sqrt(5))/2” into Google to do this computation. When you do the next part, keep this number in mind. (This number is called the Golden ratio—it is found everywhere in nature!)

Now divide the first 15 consecutive pairs of Fibonacci numbers. For example, to divide the first consecutive pair means to take $1/1$. To divide the second consecutive pair means to take $2/1$. To divide the third consecutive pair means to take $3/2$. To divide the fourth consecutive pair means to take $5/3$. And so on. If you want to use the above notation, I am asking you to find F_{n+1}/F_n for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15$.

Remark on the similarity between the first number you computed and these last numbers. For example, what do you think would happen if we took a consecutive pair really far into the sequence and divided?

Solution 2. By dividing the successive terms, we will find that the values we get are getting “closer” and “closer” to the golden ratio.

Problem 3. In terms of the problem solving strategies and techniques used, what problems have we done previously that are similar to Fibonacci’s puzzle? Relate the solution to the puzzle in terms of the PSSSP strategies. Incomplete sentences are fine here.

2 Price guessing

Problem 4. When a farm burned, they found a bill: 86 turkeys, \$.04.6.. The fire smudge obliterated the first and last digit of the total price of the birds. What are the two faded digits and what was the price of a single turkey?

Solution 3. 204.68 works. The method is just that what is used in the notes.

3 Extra credit

Problem 5. Come up with a sequence of your own and describe how to construct it (don’t use the ones in the next problem).

Problem 6. Make a new sequence by squaring the Fibonacci numbers (i.e., $1, 1, 4, 9, 25, 64 \dots$). Now add consecutive numbers of this sequence (e.g., $1 + 1, 4 + 1, 9 + 4, 25 + 9$). Relate this to the original Fibonacci sequence.