

- **Stretches**

1. **Dominoes on a Chessboard:**

You have a pile of dominoes and a chessboard. Can we cover the entire chessboard with dominoes? Now say you remove two opposite corners of the chessboard. Can you still cover the chessboard with dominoes?

Solution:

By removing opposite corners of the chess board, we see that the number of black squares does not equal the number of white squares. However, for each domino we must have one black square and one white square. Therefore, if we cover the chessboard with n dominoes, we *must* have n white squares and n black squares on the board. Therefore, we cannot cover the new chessboard with dominoes.

2. **Three smart people and five hats:**

There are 5 hats in a dark closet: 3 are red and 2 are blue. Three smart people go into the closet, put on hats, and come out. They cannot see their own hats.

The first person says, "I can see the other two peoples' hats, but I cannot determine my own."

The second guy says the same thing.

The last guy is blind, but he says, "I know exactly what color hat I'm wearing."

What color hat is the blind man wearing?

Solution:

Label the people as A, B, C where B will be the blind person. Then since A will not know what his hat color is from B and C 's hats, we conclude that if C has a red hat, B must have a blue hat. And if C had a blue hat, B may have either a red or a blue hat. Similarly, since C will not know what his hat color is from A and B , B cannot have a red hat or else C would know his hat is not red for A would know what his hat was (blue). Therefore B must have a blue hat.

- **Lecture on Be Proactive**

Today we will lecture on the strategy of being proactive. It should be pretty clear that in order to solve any given problem, you have to attack it and keep attacking it until the problem is solved. This is why to *Be Proactive* is probably the most important and going to be the most used strategy in this class. Today we will consider different ways for being proactive. The first way is to *Make a Commitment*.

- **Make a Commitment**

Studies have shown that most university students think it takes 5 minutes or less to solve a math problem. On the contrary, math problems can take anywhere from 5 minutes to decades to solve. The latter of which you will never face, but I will. Then there are problems you will face in your careers that might taken months to solve. So like any problem you face that you want to complete, it is clear you must make a commitment to solving the problem. When you are facing a math problem, you mustn't give up after a few minutes or attempts—you must keep attacking. As a demonstration, we will consider the following problem:

Changing 50 cents

In how many ways can you change a 50 cent piece?

Before we solve this problem, we have to ask ourselves several questions. The first one being, What does it mean to change a 50 cent piece?

Ask class

The second being, What sort of answer are we looking for?

Ask class

And perhaps the last, What sort of methods should we employ?

Next we shall consider another method of being proactive.

– **Get a Grip**

As we saw in the Changing 50 cents problem, we saw that there may be an issue in what exactly the question was asking. It is fairly common for questions to be ambiguous and, as a result, impossible to solve until there is disambiguation. So before solving a problem it may become important to ask yourself if you truly understand what is being asked. To demonstrate the importance of this, let's consider a fairly common encounter between teachers and students:

Student: I can't solve this problem.

Teacher: What does the problem ask?

Student: I need to find the energy required to lift something.

Teacher: What is energy?

Student: I don't know.

This is actually fairly common when tutoring physics and math and it's also fairly common that once the question is clarified, the student is able to solve the problem.

Now, to further demonstrate the importance of *Getting a Grip*, consider the following problem:

Shapes

Five circles balance six triangles. One square balances a circle and triangle together. How many squares balance eleven triangles?

There are at least two ways to interpret what this problem is asking.

Ask class what ways they think

First way: By balance we mean the weight of five circles is equal to the weight of six triangles

Second way: By balance we mean five circles physically hold up six triangles.

Solution:

Let S =squares, C =circles, and T =triangles. Then $5C = 6T$, $1S = C + T$. Thus, $C = \frac{6}{5}T$, giving $1S = \frac{6}{5}T + T = \frac{11}{5}T$. Therefore $5S = 11T$.

So now I want you to attempt to solve this problem on your own.

We now move on to the another method of being proactive.

– **Ask Questions**

This method won't be used so much in this class as perhaps it is used in real world problems. This method is to ask questions. Ask your friend. Ask your parents. Ask the internet. Just ask someone. If you don't know the answer to something, you shouldn't be ashamed of asking for help. For example, going to office hours is a very powerful tool in university. Another example is asking the IT guys about why your computer won't turn on. It's way too often that I see people giving up on problems that can be easily solved by simply googling the problem. So if you are stuck, ask questions and maybe someone can at the very least lead you to the correct solution! But, of course, you should not be asking how to solve you homework! Though it is okay to ask for help.

Now we will go over one last method for being proactive

– **Take care of your tools**

If you want to affectively solve problems, be it in math or in general, you have to be in a healthy state of mind. You need to well rested, well relaxed, and comfortable. For example, if you haven't noticed, it's damn hot in this room. I actually have a class in this room right after this class and it's brutal. So, when you need to problem solve, say during an exam or homework, try to get a good night sleep the night before. Try listening to pleasant music that won't distract you.

So that's all I have to say on being proactive for now. Hopefully the next time you come across a problem, you might be better prepared to attack and keep attacking until the problem is solved.

– **Milk and Coffee**

An 8-ounce cup of coffee and an 8-ounce cup of milk are sitting side by side. You take an ounce of milk and stir it into the coffee. Then you take an ounce of the mixture and stir it back in the milk. Is there more milk in the coffee or coffee in the milk, or is it equal?