week1lecture2

- Stretches
  1. *Cutting the String!* A piece of string is 10 inches long. What is the smallest number of scissor cuts necessary to get 10 one-inch pieces?
     - Solution: Only one cut is needed! Imagine taking scissors whose blades are half an inch thick. Wrap the string around one blade and cut.
  2. *Earrings!* In a village of 800 people, 3% wear one earring. Of the remaining 97%, half wear two earrings and half wear none. How many earrings are worn?
     - Solution: There are 800 earrings! Imagine the half of the 97% people partner up with the other half and hand their partner one earring. Then we see that there is exactly one earring per person.
- Heap of Beans
  - The first homework assignment is going to be about the Heap of Beans problem. So to be safe, we are going to go over the Heap of Beans problem both by explanation and by math. I will need someone to explain the Heap of Beans problem using as little math as possible and one student to try to explain it mathematically.
  - Non-mathematical explanation:
    The goal is to be the second player and make it so that after your turn, the pile has 4, 8, or 12 beans.
  - Mathematical explanation:
    Be the second player. Let B_1 be the number of beans player one takes at a turn. Let B_2 be the number of beans player two takes at a turn. Then, after player one and player two take their turns, we must have $B_1 + B_2 = 4$.
- *Two Bean Heaps.* Make two heaps of beans with 10 beans in each heap. You and a partner alternate moves until all the beans are gone. The winner is who takes the last bean(s). A move consists of removing one bean from one of the piles, or of removing a bean from each pile. What is a winning strategy?
- *Cycling Heaps.* Start with a collection of several heaps of beans. We create a new collection of heaps by removing one bean from each original heap, and form a new heap out of the beans removed; i.e., each original heap shrinks by one, and you create a new heap consisting of the removed beans.
  - When is the new collection of heaps the same as the original collection of heaps; e.g., if you had heaps of 2, 3, and 3 beans, then after one move you will have heaps of 1, 2, 2, and 1 beans and therefore the new collection is not the same as the original.
    * Draw examples on the board.
    * Solution: If you use the collection of heaps of sizes $1, 2, 3, \ldots, n$, the new collection will always be the same as the original.
  - Consider doing a bunch of these moves. Give an example of a collection such that after a certain amount of moves you end up with the original collection.
    * Draw examples on the board.
    * Solution: If you use the collection of heaps of sizes $2, 3, 4, \ldots, n$, it will take n moves to cycle.
- *Judge* You are a judge and two suspects are brought before you. One is a murderer and the other is not. One always lies and the other always tells the truth. What question can you ask one of the suspects to determine who is the murderer.

- Draw table on board.
- Ask one of the suspects who the other suspect will say is the murderer. Let A be the suspect you ask and B be the other suspect. If A tells the truth, then B lies. Therefore, B will tell A who is not the murderer and since A tells the truth, he will pass on this lie. If A tells a lie, then B tells the truth. Therefore, B will tell A who is the murderer and since A will lie, he will tell you who is not the murderer.