

- **Return of the Tethered Goat**

Today we will consider two problems similar to the tethered goat problem we saw last class. We will consider two variations of the problem and focus on how we would approach drawing the figure and writing up a solution.

Variation 1

Suppose we have a funky equilateral triangular barn of side length 10ft. Tie a goat to one corner with a 15ft rope. What is the square footage of the goat's roaming?

Factors to consider:

Then angle of each corner of an equilateral triangle is 60° .

We can cut 6 slices of 60° of a circle.

$$A = \pi r^2$$

The important part is to write up a really explanatory solution to the problem. Imagine you are trying to teach this to a child, or your parents. There is no harm in explaining too much. After this, I will have one or two people come up and convince us you have a solution and your solution works. This way you will get practice in explaining how to solve a problem, rather than just solving it. In the real world, when you are presented a problem, you are going to have to explain why your solution works. No one will trust you or take you seriously if you just present an answer—you need to convince them you are correct.

Variation 2

Suppose we remove the wall opposite to the corner the goat is tied to. What is now the square footage of the goat's roaming?

- **Prom Problem**

We will now go over what the next homework assignment is. This one will use the visualization technique of tables and a little bit of logic. You will always need to use lists and figures. The logic, however, will be basically using process of elimination. This assignment will be due next Thursday and I recommend turning in a hard copy. The following problem is in the text, but I changed the problem a little so the problem isn't culturally outdated.

Prom Problem

After the senior prom, six friends went to their favorite restaurant, where they share a booth. The group consisted of the senior class president, the valedictorian, the head cheerleader, a player on the school volleyball team, a player on the school basketball team, and the school principal's only child. Their names were Bobby, Frank, George, Jazz, Rachel, and Sally, not necessarily in that order. Bobby, George, and Sally have red hair. Frank, Jazz, and Rachel have black hair. Each of the six was in love with one of the others of the opposite hair color, but no two had crushes on the same person.

- Frank liked the cheerleader but was sitting opposite the valedictorian.
- George was sitting next to the cheerleader and adored the class president.
- Bobby was in love with the person sitting opposite her.
- Jazz, who was not the valedictorian, was sitting between the volleyball player and the class president.
- Rachel disliked the basketball player.
- Sally, whose parents worked at a bank, was sitting against the wall and had a crush on the volleyball player.
- The volleyball player sat opposite the principal's child.

Do we need any clarifications? A booth is the following.

- **Rest of the class**

For the rest of the class we will go over two problems on the board. These problems are all of the sort that have a “trick” to them. They are not the type you can brute force to solve. But they are pretty cool problems that I learned when I was studying a more rigorous version of problem solving. The point of these problems are to show that sometimes you may get stuck with an incorrect idea for a solution. Most people are not able to solve this problem without thinking about them for a long time, so will spend only a few minutes working on it and then I will show you the answer.

The first problem is the following.

Man on the Mountain

A man leaves home for a mountain at 1pm and reaches the top at 3pm. The next day he departs from the top at 1pm and gets home at 3pm, by following the same path as the day before. Was he necessarily ever at the same point on the path at the same time on both days?

Solution:

Imagine one person starting on the top of the mountain and another person starting at the bottom of the mountain. They will necessarily run into each other. We see that no matter how slow, or how fast either walk, they will meet at the same point at some specific time.

Back and Forth Fly

Two trains are 200 miles apart and perilously heading towards each other on the same track each travelling at 50 miles per hour. A fly is located on the most forward part of the train of one of the trains and flies away straight towards the oncoming train at 80 miles per hour. When it reaches the other train, it turns around just in time to avoid getting hurt and flies back towards the first train. It does this back and forth until it can fly no more. How far will the fly have travelled.

Solution:

This problem is tricky because you feel forced to focus on finding the distance travelled each time the fly turns around. However, the problem is equivalent to finding how long it takes the two trains to meet. This is because this is exactly how long the fly will have flown. Therefore, since it will take 4 hours for the trains to collide, the fly will have flown $80\text{mph} * 4 = 320\text{miles}$.