1 Stretches

1. Rearrange the letters in the words *new door* to make one word.
2. Put ten horses in 9 stalls.
3. A cube of wood 3 inches on each edge is to be cut into cubes 1 inch on an edge. After each cut with a (very thin) saw, the pieces may be piled in any manner you wish before the next cut. What is the smallest number of cuts required?

2 Problems

Problem 1. *You have placed in storage 300 pounds of cucumbers which are 99% by weight. After a week or so, some evaporation has occurred whereby the cucumbers are now only 98% water. What does your batch of cucumbers now weigh?*

Solution 1. At first, the problem seems to have an intuitive answer, or at least it feels like you can grasp how much a of difference in weight there should be. A pretty understandable guess would be to guess the cucumber batch now weighs just a few pounds less than it did starting out. We will show that this initial intuition will fail us and we need to build a new intuition.

1. Let's start off by finding the initial weight of solid in this batch.
   If the batch is 300 pounds and 99% water by weight, then 1% is solid weight and so we have 0.01*300=3 pounds of solid cucumber.
2. After evaporation, the batch is 98% water by weight and so 2% solid by weight.
   Let W be the final weight after evaporation.
   Then 0.02W=3 pounds, since we haven't lost any solid weight.
   Therefore, the final weight is W=3/0.02=150 pounds.
   Note how vastly different this is from our intuition.

Problem 2 (Multiple Locks)**.** *You belong to a Math Club that has accumulated valuable jewelry. The treasure is kept in a chest. For security reasons, you want at least two of your 10 club members present when the chest is open. To insure this, you put multiple locks on the chest and distribute keys to members so that no one member can open the chest, but any two members can. Each lock has a different key, but you can make several copies of the same key to distribute to the club members. What is the fewest number of locks and keys that you will need?*

Solution 2.

1. This problem may seem difficult to parse at first so let's apply our Pause and Reflect strategy and ask ourselves what is actually being asked. Things to assume:
   - Each lock has only one key that opens it.
   - No single lock has two or more keyholes in it.
   - Members of the club cannot trade keys.

   What is the unknown? The fewest number of locks and keys that will be needed.
2. We see that this problem may be too difficult to attack without simplification. Since we all have mastered the simplification strategy, let's try this! We see that there are mainly two ways to solve the problem, namely by simplifying the number of students and simplifying the number of locks. So let's do both!
3. Let's do several cases of simplification.
   - If there was just a single member, what is the minimum number of locks? Well this is kind of a trick question as we need at least two member for our conditions to be met.

- What about two members?
  If any two members can get in but no individual can, a single lock will not work.
  So let's consider two locks called A and B.
  If we give member 1 a key to lock A and member 2 a key to lock B, then we see this works.

- What about three members?
  - We will see two locks cannot work.
    In this case, we'd have to give each student at most one.
    For example, 1 and 2 get a key for A and 3 gets a key for B.

  - What about three locks? Each member cannot get three keys.
    Each member cannot get only one key.
    Each pair of members must have all three keys collectively, so no two members can have the same set of keys.
    Consider if 1 had only one key A.
    Then 2 must have keys B and C.
    Then 3 must have keys A and C or A and B.
    Thus 1 and 3 cannot unlock the chest so each member must have 2 keys.
    How to give the keys?
    If 1 had keys A,B, then 2 must have keys A,C or B,C.
    Suppose 2 has A,C, then B must have B,C.
    Suppose 2 has B,C, then B must have A,C.

4. At this stage, let's do apply the Pause and Reflect strategy. Do we see a pattern emerge?
   Some things to consider:
   - If there are n people, there are n locks.
   - If there are n people, each person gets $n-1$ locks.
   - Another way of looking at this: each member gets all keys but 1 and each member lacks a different key.
   - So we have
     (a) Since each member lacks a key, the member cannot unlock the chest.
     (b) Since each member lacks a different key, each pair of members will collectively have all the keys to unlock the chest.
5. Conclusion: With 10 members, we must have 10 locks and 90 keys since each member gets 9 keys.