1 Problems

Problem 1 (Jumping Frogs)**.** *How many ways can a frog hop up a twelve-step staircase if the frog can hop either one or two steps on each hop?*

Solution 1. It is oftentimes useful for understanding a problem like this to just list out a bunch of examples. Doing so may allow us to understand the problem a little more than we did starting off. It may give us a "feel" for what the solution should look like. So go ahead and write out a couple of different ways a frog can hop up the twelve-step staircase.

We see that there are *a lot* of different ways this can be done. This problem is a bit like *Changing 50 cents* in that there is a lot of data to keep track of.

Let's relate this so the *Pause and Reflect* by breaking the problem down to three parts: In the Beginning, In the Middle, and In the End. **In the Beginning**
Based on our sampling, What *should* our solution look like? We see that we *should* get a fairly large number.
Can we *Simplify it*? We can simplify the problem in at least one way: we can consider a smaller number of steps.
With this in mind, can we *See it* more clearly? There are perhaps two tools we can use to *See it* more clearly: draw a picture of the situation and construct a table to organize information.
Can we recall a similar problem? This problem isn't quite a variation of anything you've seen before, but it is perhaps similar to the problem about locks and similar problems that deal with this counting type of argumentation.
Lastly, can we come up with an effective plan of attack? In either case, let's try to come up with a plan of attack!
Plan of Attack

1. *Simplify* the problem.
   Let's consider fewer number of steps for the ladder to see if we can develop a pattern.
2. *See it* by constructing a table labeled with *(# of hops, # of ways)*.
3. Fill out table with the "easier" cases.
4. Ask ourselves if there is a pattern and if we are progressing towards a solution.
5. Solve the problem if we can.

So let's jump right in and *Be Proactive* by attacking the problem with our plan of attack!

1. As we said, we can problem simplify the problem by considering fewer steps.
   (a) Try 1 step.
   Well, clearly there is only one way as the frog can only hop one step here.
   Let's write (1) to mean do one step.
   (b) How about 2 steps?
   The frog can do 2 single-step hops and 1 two-step hops.
   Let's write (1,1) to mean do 2 single-step hops and (2) to mean do 1 two-step hops.
   (c) How about 3 steps?
   We have (1,1,1), (1,2), and (2,1), so three.
   (d) How about 4 steps?
   We have (1,1,1,1), (1,1,2), (1,2,1), (2,1,1), and (2,2), so 5.
   (e) How about 5 steps?
   We have (1,1,1,1,1), (1,1,1,2), (1,1,2,1), (1,2,1,1), (2,1,1,1), (1,2,2), (2,1,2), (2,2,1), so 8.
   (f) Let's do one more with 6 steps.
   We have (1,1,1,1,1,1), (1,1,1,1,2), (1,1,1,2,1), (1,1,2,1,1), (1,2,1,1,1), (2,1,1,1,1), (1,1,2,2), (1,2,1,2), (2,1,1,2), (1,2,2,1), (2,1,2,1), (2,2,1,1), (2,2,2), so 13.

2. Let's construct the table as planned.

3. Now let's fill out the table using the data we gathered.

4. Can we see a pattern? Hopefully you recognize the most amazing sequence know as Fibonacci's sequence!

In the Middle
Based on our work so far, let's ask some relevant questions.
Does it appear we are stuck? Nope.
Have we made any progress? It sure seems so!
Have we made any mistakes or the like?

So from here it sure seems we have a solution to the problem. So far it seems the Fibonacci sequence can give us the solution to the problem and so we just need to write the Fibonacci sequence and count off 12 numbers starting off at the second 1. That is, since the sequence is 1,1,2,3,5,8,13,21,34,55,89,144,233, the twelfth number from the second 1 is 233. However, this is technically not sufficient as we don't absolutely know this pattern will always hold. Let's try to explain this issue away.

Let's explain this by example. We will see that we can get the 6 step case by the 5 and 4 step cases. To go from 5 step case to 6 step case, we simply add a 1 to the end of each 5 step case. To go from a 4 step case to a 6 step case, we simply add a 2 to the end of each 4 step case. That is, the number of 6 step cases is the number of 4 and 5 step cases combined.

In the End Let's try to see if we can learn anything from this problem. Note that some of the questions I talked about when we discussed the *Pause and Reflect* strategy cannot really be answered here since they require you to first struggle with the problem. However, the main goal would be to first take note at the general method of doing the table, which you should be familiar by now. Secondly, take note of the last thing we discussed.