1.2 Mathematical Modeling

Mathematical Modeling

0.1 Definitions/Terms

Definition 1.

1. ***Fixed Cost**: cost that does not depend on amount of products being manufactured.*
2. ***Variable Cost**: Does depend on the amount of products being manufactured.*
3. ***Cost**: cost = (variable cost) + (fixed cost).*
4. ***Linear Cost Model**: Let $x =$# of units of a given product ($x \geq 0$), m=cost per one unit (fixed). Then variable cost = (cost per unit) $\times$ (# of units produced) = mx. Let b be the fixed cost and $C(x)$ be the cost. Then*

$$\begin{aligned} C(x) &= \textit{cost} \\ &= \textit{variable cost + fixed cost} \\ &= mx + b. \end{aligned}$$

5. ***Linear Revenue Model**:Let $p =$price of units sold (fixed), $x =$number of units sold (assume = # of units produced, and let $R(x)$ be the revenue. Then*

$$\begin{aligned} R(x) &= \textit{revenue} \\ &= \textit{(price per unit)} \times \textit{(\# sold)} \\ &= px. \end{aligned}$$

6. ***Profit**: Let $P =$profit. Then*

$$\begin{aligned} P &= \textit{profit} \\ &= \textit{(revenue)-(cost)} \\ &= R - C. \end{aligned}$$

7. ***Break Even Quantity**: This is the quantity x for which $R(x) = C(C)$.*

0.2 Examples

Example 1 (Cereal Manufacturing Industry)**.** Estimates shows a cereal manufacturer obtained a price of \$4800 a ton of cereal.

- Fixed costs for a typical plant is \$300 million to run
- Total variable cost is \$3840/ton.

Find costs, revenue, and profit equations, and find the break even point.

Mathematical Models of Supply and Demand

0.3 Definitions/Terms

Definition 2. 1. ***Demand Equation**: Let $x =$# of units produced by entire industry. Expect as x increases, p will go down since there will be less of a demand for product. We then making the assumption $p = -cx + d$, $c > 0$, to model the price. This equation is called the demand equation and its graph the demand curve.*

2. ***Supply Equation***: *The equation $p = p(x)$ gives the price p necessary for suppliers to make x unites available to the market. The equation is called the supply equation and its graph the supply curve.*

3. ***Equilibrium Point***: *When supply equals the demand; i.e., when demand and supply curves intersect. At this point, $x-$coord=equilibrium quantity, $y-$coord=equilibrium price.*

0.4 Examples

Example 2 (Finding Equilibrium Point)**.** Tauer determined demand and supply curves for milk in this country. If x is in billions of pounds of milk and p is in dollars per hundred pounds, he found that the demand function for milk was $p = 56 - 0.3x$ and the supply function was $p = 0.09x$. Find the equilibrium point.

Quadratic Mathematical Models

Definition 3 (Vertex Point)**.** *Let $q(x) = ax^2 + bx + c$ be a quadratic. Then $q(x)$ can be written in the form $a(x - h)^2 + k$ for some constants h and k. The point (h, k) is called the vertex and ca be found using the formulae:*

$$h = -\frac{b}{2a} \qquad k = c - \frac{b^2}{4a}.$$

Note: *If $a > 0$, the graph of q is concave up and so q assumes a minimum of k when $k = h$. If $a < 0$, the graph of q is concave down and so q assumes a maximum of k when $x = h$.*

Example 3 (A Quadratic Revenue Function)**.** In 1973 Braley and Nelson studied the effect of price increases on school lunch participation in the city of Pittsburgh. They estimated the demand equation for school lunches to be $p = -0.00381x + 62.476$, where x is the number of lunches purchased and p is the price in cents. Find the value of x for which the revenue will be maximum, and find the maximum revenue. For this value of x, find the price.