

In this document is a list of common mistakes that either I have committed or I have seen committed by students.

Notice how a lot of these mistakes can be avoided by simply doing some sort of test computation.

Mistake 1. Let f be some function. Then $f(x+h) = f(x) + f(h)$.

Explanation 1. This is not true in general! You **cannot** distribute f like this! You must evaluate f at $x+h$. For example, if $f(x) = x^2$, note that $f(x+h) = (x+h)^2 = x^2 + 2xh + h^2$ and $f(x) + f(h) = x^2 + h^2$. That is, $f(x+h) \neq f(x) + f(h)$ here.

Note that if $f(x) = x$, then $f(x+h)$ does in fact equal $f(x) + f(h)$, so the mistake is that $f(x+h) \neq f(x) + f(h)$ in *general*.

Mistake 2. $(a+b)^2 = a^2 + b^2$ or $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$.

Explanation 2. This is the same mistake as saying $f(a+b) = f(a) + f(b)$ in general. Let's just compute an example to see this isn't true.

For example, $(3+3)^2 = 6^2 = 36$, yet $3^2 + 3^2 = 9 + 9 = 18$, and we all know $18 \neq 36$. Similarly, $\sqrt{2+2} = \sqrt{4} = 2$, yet $\sqrt{2} + \sqrt{2} = 2\sqrt{2} \neq 2$.

Mistake 3. $\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = h \left(\frac{1}{x+h} - \frac{1}{x} \right)$

Explanation 3. This mistake is usually just due to forgetting how to divide rational expressions. The correct simplification is

$$\frac{\frac{1}{x+h} - \frac{1}{x}}{h} = \left(\frac{1}{x+h} - \frac{1}{x} \right) \frac{1}{h} = \frac{1}{h(x+h)} - \frac{1}{hx}.$$

Mistake 4. $\frac{1}{x+a} - \frac{1}{x} = \frac{a}{x(x+a)}$.

Explanation 4. This mistake can be seen as either dropping a negative or neglecting to distribute the negative to a . The correct simplification is

$$\frac{1}{x+a} - \frac{1}{x} = \frac{x}{x(x+a)} - \frac{x+a}{x(x+a)} = \frac{x - (x+a)}{x(x+a)} = \frac{x - x - a}{x(x+a)} = \frac{-a}{x(x+a)}.$$

Notice how we *needed* to distribute the negative to a in $x - (x+a)$.

Mistake 5. $\frac{1}{a+b} = \frac{1}{a} + \frac{1}{b}$.

Explanation 5. When we have a fraction like this, we cannot split the fraction like this! Let's test it. If we could, then $\frac{1}{2} = \frac{1}{1+1} = \frac{1}{1} + \frac{1}{1} = 1 + 1 = 2$, which is certainly wrong!

Mistake 6. $\frac{f(x+h)-f(x)}{h} = \frac{f(x)-f(x)}{=0}$ or $\frac{f(x)}{x} = f(1)$.

Explanation 6. This mistake is probably just due to misunderstanding how functions work. In any case, you cannot just "pull things" out of the functions input like this. Whenever you have a function f , you must evaluate $f(x+h)$ *first*, then divide by h and so on. Similarly for $f(x)/x$, you must evaluate $f(x)$ first, then divide by x .

Mistake 7. $\frac{0}{0} = 1$.

Explanation 7. You can *never* divide by zero. Having zero in the numerator and zero in the denominator does not allow you to use cancellation; $\frac{0}{0}$ is just not defined whatsoever.