

4.1 Derivatives of Powers, Exponents, and Sums

Properties of Derivatives

1. If c is any real number, then

$$\frac{d}{dx}(c) = 0$$

2. If a is any real number, then

$$\frac{d}{dx}(x^a) = ax^{a-1}$$

Note that a may be positive, negative, an integer, a fraction, an irrational, etc.

3. If c is any real number and $f'(x)$ exists, then

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

4. The derivative of e^x is e^x :

$$\frac{d}{dx}(e^x) = e^x$$

5. If $x > 0$, then

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

6. If $f'(x)$ and $g'(x)$ exist, then

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

7. If $f'(x)$ and $g'(x)$ exist, then

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$

8. If $f'(x)$ and $g'(x)$ exist, then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

if $g(x) \neq 0$.

Examples

Example 1. Let $f(x) = 4$ and find $f'(x)$.

Solution 1. Since $f(x)$ is a constant function, we get immediately $f'(x) = 0$.

Example 2. Find the derivatives of the following functions

(a) $x^2 + 1$

(b) $x^{3.2} + x^{-4}$

(c) $2/x^2 + x$.

Solution 2. cf. lecture for solutions.

(a) $x^2 + 1 \rightarrow 2x$

(b) $x^{3.2} + x^{-4} \rightarrow 3.2x^{2.2} - 4x^{-5}$

(c) $2/x^2 + x \rightarrow -4/x^3 + 1$

Example 3. Find the derivatives of the following functions

(a) $e^x + x + e$

(b) $\ln(x) + x^2 + 2e^x + \pi$

(c) $x^3(\sqrt{x} + x^{1/4}) + \ln(x) + e^\pi$

Solution 3. cf. lecture for solutions.

(a) $e^x + x + e \rightarrow e^x + 1$

(b) $\ln(x) + x^2 + 2e^x + \pi \rightarrow 1/x + 2x + 2e^x$

(c) $x^3(\sqrt{x} + x^{1/4}) + \ln(x) + e^\pi \rightarrow 4.5x^{3.5} + 3.25x^{2.25} + 1/x$

Example 4.