Q Center Exam 2 Review Solutions:

(1) First, note domain of f is $(0, \infty)$

Now, $f'(x) = 1 - \frac{1}{x}$

Setting $f'(x) = 0$, we get $\boxed{x = 1 \text{ as a critical value}}$

(*) Side Note: $f'(0)$ DNE, but $x = 0$ is not a C.V since 0 is not in the domain.

To Find where f is increasing/decreasing:

f'(test)		$f'(.5) = -$		$f'(2) = +$	
f	0	decreasing	1	increasing	∞

So f increasing on $(1, \infty)$

decreasing on $(0, 1)$

And have a rel min at $x = 1$

(2) First, domain of g is $(-\infty, \infty)$

$g'(x) = 4x^3 - 12x^2 + 8x$

$g''(x) = 12x^2 - 24x + 8$

Set $g''(x) = 0$ using quadratic formula.

get $x \approx 1.58$ or $x \approx 0.42$

$g''(\text{test})$ | $g''(0) = +$ | $g''(1) = -$ | $g''(2) = +$

$-\infty$ | 0.42 | 1.58 | $+\infty$

$\cup$ $\cap$ $\cup$

g conc up | conc down | conc up

So g is concave up on $(-\infty, 0.42\ldots) \cup (1.58\ldots, \infty)$

and concave down on $(0.42\ldots, 1.58\ldots)$

w/ inflection pts at $x \approx 0.42$ and $x \approx 1.58$

(3) a) $h'(x) = 35(3x - e^x)^{34} \cdot (3 - e^x)$

b.) $m'(x) = (\ln(x)) \cdot \left[3(1+6x^2)^2 \cdot (12x)\right] + (1+6x^2)^3 \cdot \dfrac{1}{x}$

c) low $= 2x^3$ dlow $= 6x^2$

hi $= x + e^x$ dhi $= 1 + e^x$

$\rightarrow$ $y'(x) = \dfrac{2x^3 \cdot (1+e^x) - (x+e^x) \cdot 6x^2}{(2x^3)^2}$

4) $f(x) = \dfrac{1}{x^2}$, $f(x+h) = \dfrac{1}{(x+h)^2} = \dfrac{1}{x^2+2x+h^2}$

$$f'(x) = \lim_{h \to 0} \frac{\dfrac{1}{x^2+2xh+h^2} - \dfrac{1}{x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{x^2 \cdot 1}{x^2 \cdot (x^2+2xh+h^2)} - \dfrac{(x^2+2xh+h^2) \cdot 1}{(x^2+2xh+h^2) \cdot x^2}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{x^2}{x^2(x^2+2xh+h^2)} - \dfrac{x^2+2xh+h^2}{x^2(x^2+2xh+h^2)}}{h}$$

$$= \lim_{h \to 0} \frac{\dfrac{-2xh-h^2}{x^2(x^2+2xh+h^2)}}{h} = \lim_{h \to 0} \frac{\cancel{h}(-2x-h)}{x^2(x^2+2xh+h^2)} \cdot \frac{1}{\cancel{h}}$$

$$= \frac{-2x-0}{x^2(x^2+0+0)} = \frac{-2x}{x^4}$$

$$= -\frac{2}{x^3}$$

(5) Since $\lim_{x \to 3^-} p(x) = -\infty$ and $\lim_{x \to 3^+} p(x) = +\infty$

- $\lim_{x \to -3^-} p(x) = +\infty$ and $\lim_{x \to -3^+} p(x) = -\infty$

There are vertical asymptotes at $x = 3$ and $x = -3$

Since $\lim_{x \to +\infty} p(x) = 2$ and $\lim_{x \to -\infty} p(x) = 2$

There is a horizontal asymptote at $y = 2$

---

(6) $p(x) = \dfrac{x+1}{x-1}$

<u>Asymptotes</u>: • vertical at $x = 1$ Since $\lim_{x \to 1^-} p(x) = -\infty$ $\lim_{x \to 1^+} p(x) = +\infty$

• since $\lim_{x \to \pm\infty} p(x) = 1 \Rightarrow$ horizontal at $y = 1$

<u>Increase/decrease</u>: $p'(x) = \dfrac{-2}{(x-1)^2}$ $p(x)$ decreases on $(-\infty, 1) \cup (1, \infty)$

<u>Concavity</u>: $p''(x) = \dfrac{4}{(x-1)^3} \Rightarrow$ $p(x)$ concave up on $(1, \infty)$

concave down on $(-\infty, 1)$