# Differentiation

- Power rule: $\frac{d}{dx}x^a = ax^{a-1}$ for **any** constant $a$.
- Summation rule: $(g+h)' = g' + h'$.
- Scaling rule: $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$.
- Product rule: $(gh)' = g'h + hf'$.
- Quotient rule:$\left(\frac{g}{h}\right)' = \frac{g'h - gh'}{h^2}$.
- Derivative of $e^x$: $\frac{d}{dx}e^x = e^x$.
- Derivative of $\ln x$: $\frac{d}{dx}\ln x = \frac{1}{x}$.
- Derivative of a constant: $\frac{d}{dx}c = 0$ for all constants $c$.
- Chain rule: $\frac{d}{dx}h(g(x)) = g'(x)h'(g(x))$. You may also wish to memorize the other chain rules. However, all of those are derived from this chain rule; i.e., this single chain rule will always work. The only weird cases are

$$\frac{d}{dx}a^{f(x)} = f'(x)a^{f(x)}\ln a \qquad \text{and} \qquad \frac{d}{dx}\log_a|f(x)| = \frac{f'(x)}{f(x)\ln a}.$$

- Limit definition of the derivative: $f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$. Know how to compute this limit for functions that look like $x$, $x^2$, $\frac{1}{x}$ and $\frac{1}{x^2}$. Note that this include variations of such functions: e.g., $x^2+2$, $\frac{1}{x-1}$, etc.
  **Note also that if the question asks to use the limit definition, use the limit definition.** If you use **any** other method, you will not be given **any** credit.

# Tangent lines

Know how to find the equation of a tangent line given a function $f$. In general, the tangent line of $f$ through the point $(c, f(c))$ is given by $y_{\tan} = f'(c)(x-c) + f(c)$. The questions are usually posed as, Find the tangent line of $f$ at $x = c$.

# Horizontal Asymptotes and limits at infinity

Make sure you can compute $\lim_{x\to+\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ for the following types of cases:

1. $f(x) = \frac{x^2+x}{3x^5+2x-1}$ (degree on bottom is larger).
2. $f(x) = \frac{-x^7+2x^3+1}{x^3+x+1}$ (degree on top is larger).
3. $f(x) = \frac{x^5+2x+4}{2x^5+1}$ (degrees on top and bottom are equal).
4. Any of the previous types but with exponentials thrown in; e.g., $f(x) = \frac{e^{-2x}+x}{e^{-5x}+x}$.

## Applications of the constant sign theorem

- Know how to find the critical points of a function.
- Know how to use the constant sign theorem to determine where a function is increasing or decreasing. That is, determine where $f' > 0$ or $f' < 0$.
- Know how to use the constant sign theorem to determine where a function is concave up or concave down. That is, determine where $f'' > 0$ or $f'' < 0$.
- Use the previous item to determine what are the inflection points.

## Derivative tests

- Use the first derivative test to determine whether or not a critical point $f'(c) = 0$ is a relative extremum or not, and if it's a relative maximum or minimum if it's a relative extremum.
- Use the second derivative test to determine if a critical point $f'(c) = 0$ is a relative extremum, and recognize that if $f''(c) = 0$, then the second derivative test fails to tell you anything.

## Reading a graph

This covers the graph type questions from the homework. Note that being good at this will help with curve sketching. Examples are: where $f$ increasing, decreasing, concave up, concave down, extrema, where is $f$ not differentiable, etc. Note that you may be given a graph of the function $f$ or its derivative $f''$.

Where is a function not differentiable: This includes discontinuities, vertical asymptotes, pointy places, where the graph is vertical (and hence has a vertical tangent line), etc.

## Curve sketching

Be able to know how to, when given a list of information about the function (e.g., where its derivative is positive, where $f'(c) = 0$, etc.), how to approximately sketch the corresponding graph.