

## 4.1 Derivatives of Powers, Exponents, and Sums

### Properties of Derivatives

1. If  $c$  is any real number, then

$$\frac{d}{dx}(c) = 0$$

2. If  $a$  is any real number, then

$$\frac{d}{dx}(x^a) = ax^{a-1}$$

Note that  $a$  may be positive, negative, an integer, a fraction, an irrational, etc.

3. If  $c$  is any real number and  $f'(x)$  exists, then

$$\frac{d}{dx}(cf(x)) = c\frac{d}{dx}(f(x))$$

4. The derivative of  $e^x$  is  $e^x$ :

$$\frac{d}{dx}(e^x) = e^x$$

5. If  $x > 0$ , then

$$\frac{d}{dx}(\ln x) = \frac{1}{x}$$

6. If  $f'(x)$  and  $g'(x)$  exist, then

$$\frac{d}{dx}(f(x) \pm g(x)) = \frac{d}{dx}(f(x)) \pm \frac{d}{dx}(g(x))$$

7. If  $f'(x)$  and  $g'(x)$  exist, then

$$\frac{d}{dx}(f(x)g(x)) = f(x)\frac{d}{dx}(g(x)) + g(x)\frac{d}{dx}(f(x))$$

8. If  $f'(x)$  and  $g'(x)$  exist, then

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

if  $g(x) \neq 0$ .

### Examples

**Example 1.** Let  $f(x) = 4$  and find  $f'(x)$ .

**Solution 1.** Since  $f(x)$  is a constant function, we get immediately  $f'(x) = 0$ .

**Example 2.** Find the derivatives of the following functions

(a)  $x^2 + 1$

(b)  $x^{3.2} + x^{-4}$

(c)  $2/x^2 + x$ .

**Solution 2.** cf. lecture for solutions.

(a)  $x^2 + 1 \rightarrow 2x$

(b)  $x^{3.2} + x^{-4} \rightarrow 3.2x^{2.2} - 4x^{-5}$

(c)  $2/x^2 + x \rightarrow -4/x^3 + 1$

**Example 3.** Find the derivatives of the following functions

(a)  $e^x + x + e$

(b)  $\ln(x) + x^2 + 2e^x + \pi$

(c)  $x^3(\sqrt{x} + x^{1/4}) + \ln(x) + e^\pi$

**Solution 3.** cf. lecture for solutions.

(a)  $e^x + x + e \rightarrow e^x + 1$

(b)  $\ln(x) + x^2 + 2e^x + \pi \rightarrow 1/x + 2x + 2e^x$

(c)  $x^3(\sqrt{x} + x^{1/4}) + \ln(x) + e^\pi \rightarrow 4.5x^{3.5} + 3.25x^{2.25} + 1/x$

**Example 4.**