Sketching Curves procedure

Included here are steps for plotting the graph of a function when given the function.

(A) Using the function f itself.

  (a) Determine where f is continuous, its domain, and where it is positive or negative.

  (b) Is it symmetric about the y-axis?
  If $f(x) = f(-x)$, then it is symmetric and so you need only worry about $x \geq 0$ and plot for all x by using symmetry.

  (c) What are its vertical asymptotes.
  This is where the function shoots off to $\pm\infty$ at finite values of x (e.g., $f(x) = 1/x$ has a vertical asymptote at $x = 0$).

  (d) What are its horizontal asymptotes.
  These are the $y-$values $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ when they are finite (e.g., $f(x) = 1/x \to 0$ as $x \to \infty$ and so has a horizontal asymptote of $y = 0$).

  (e) The x and y intercepts.

(B) Using f'.

  (a) Find the critical points of f.

  (b) Find the intervals of increasing and decreasing by finding where $f' > 0$ and $f' < 0$, respectively.

  (c) Use this information to find any relative extrema.

(C) Using f''.

  (a) Find where f is concave up or down by finding where $f'' > 0$ or $f'' < 0$, respectively.

  (b) Find the inflection points of f; i.e., where f goes from concave up to down, or concave down to up.

(D) Use this information to plot an estimation of what the graph of f might look like.

Sketching Curves example

Example 1. We will consider the case $f(x) = (x + 1)e^x$.

(A) Using the function f itself.

  (a) The function is continuous everywhere and its domain is everywhere. To find where f is pos/neg, we use the constant sign theorem. Note that $f(x) = 0$ only when $x = -1$ and so the intervals we consider are $(-\infty, -1)$ and $(-1, \infty)$. We find $f(-2) < 0$ and $f(0) > 0$ and so f is neg on $(-\infty, -1)$ and pos on $(-1, \infty)$.

  (b) We note that $f(-1) = 0$ yet $f(1) = 2e$ and so f is not symmetric.

  (c) This function has no vertical asymptotes.

  (d) We know that $\lim_{x\to\infty}(x + 1)e^x = \infty$ and $\lim_{x\to-\infty}(x + 1)e^x = \lim_{x\to\infty}(-x + 1)e^{-x} = 0$, so the only horizontal asymptote of f is $y = 0$.

  (e) We note $f(x) = 0$ only at $x = -1$ and $f(0) = 1$, and so the x intercept is $x = -1$ and the y intercept is $y = 1$.

(B) Using f'. We find $f'(x) = (x + 2)e^x$.

  (a) The critical points are when $f'(x) = 0$; i.e., when $x = -2$ only.

(b) By the constant sign theorem we need only consider the intervals $(-\infty, -2)$ and $(-2, \infty)$. We find $f'(-3) < 0$ and $f'(0) > 0$ so that $f' < 0$ (and thus f is decreasing) on $(-\infty, -2)$ and $f' > 0$ (and thus f is increasing) on $(-2, \infty)$.

(c) It follows from the first derivative test that $f(-2)$ is a relative minimum and the only relative extremum.

(C) Using f''. We find $f''(x) = (x + 3)e^x$.

(a) We note that f'' is continuous and defined everywhere and $f''(x) = 0$ only at $x = -3$. Therefore, by the constant sign theorem, the only intervals we need concern ourselves with are $(-\infty, -3)$ and $(-3, \infty)$. We find $f''(-4) < 0$ and $f''(0) > 0$ so that $f'' < 0$ (and thus f is concave down) on $(-\infty, -3)$ and $f'' > 0$ (and thus f is concave up) on $(-3, \infty)$.

(b) This information tells us that f has an inflection point at $x = -3$ since $f''(-3) = 0$, f is concave down just left of $x = -3$, and f is concave up just right of $x = -3$.

(D) Provided in class.