Algebra Review

The goal of this section is to learn or remind you of some bits and pieces of math that will be necessary for the entire course. It cannot be stressed enough the importance of making sure you understand the review material.

Exponents

Here we shall consider some fundamental operations on real numbers. The first operation is that of taking the exponential power n (for some integer n) of some number x. So let's start with some definitions.

Definition 1 (Integer Exponents)**.** Given a real number x, we define

$$x^n = x \cdot x \cdots x,$$

where n is a positive integer and there are n factors of x in this product. Moreover, if $x \neq 0$, then we take $x^0 = 1$ and note 0^0 is not defined. Lastly, we define $x^{-n} = \frac{1}{x^n}$.

Note that this definition applies to *any* real number x; i.e., x may be an integer, it may be a fraction, it may be negative, it may be π, and so on.

Example 1. We compute the following.

1. $3^3 = 3 \cdot 3 \cdot 3 = 27$
2. $\left(-\frac{1}{\pi}\right)^0 = 1$
3. $\left(\frac{1}{4}\right)^2 = \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$
4. $4^{-2} = \frac{1}{4^2} = \frac{1}{4 \cdot 4} = \frac{1}{16}$

△

We will note find much use in seeing exponents in this light. To really see how exponents are used in math, we will often stay in variable land and refrain from explicitly using numbers. So, to see how this works, we list some properties of exponents.

Theorem (Properties of Exponents)**.** *Let m and n be positive integers. Then*

1. $x^m x^n = x^{m+n}$
2. $(x^m)^n = x^{mn}$
3. $(xy)^n = x^n y^n$
4. $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$
5. $\frac{x^m}{x^n} = x^{m-n}$ *if* $x \neq 0$
6. $\frac{x^m}{x^n} = \frac{1}{x^{n-m}}$ *if* $x \neq 0$.

As mentioned, the power here is not really in the ability to apply these properties to numbers, but rather to variables. For example, say we want to work with $(x^2y^3z^2)^3$ for whatever reason. Sometimes it will be easier to work with the form $x^6y^9z^6$, which may be obtained by applications of the rules above:

$$\begin{aligned}(x^2y^3z^2)^3 &= x^{2\cdot 3}y^{3\cdot 3}z^{3\cdot 3}\\ &= x^6y^9z^6.\end{aligned}$$

The following is a more relevant computation.

Example 2. *Simplify* $\left(\frac{x^{-2}y^2}{x^{-4}y^5}\right)^{-3}$ *and express the answer using no negative exponents.* Note that there are several ways of solving this problem, so what follows in one of (possibly) many ways. We will first apply the rule that allows us to distribute the -3 exponent.

$$\begin{aligned}\left(\frac{x^{-2}y^2}{x^{-4}y^5}\right)^{-3} &= \frac{\left(x^{-2}y^2\right)^{-3}}{\left(x^{-4}y^5\right)^{-3}} \\ &= \frac{x^{-2\cdot -3}y^{2\cdot -3}}{x^{-4\cdot -3}y^{5\cdot -3}} \\ &= \frac{x^6y^{-6}}{x^{12}y^{-15}}.\end{aligned}$$

At this point, there are shortcuts to getting to the problem immediately, but it is better to be safe than sorry! So let's just apply the rules blindly to obtain an expression with only one x and only one y:

$$x^{6-12}y^{-6-(-15)} = x^{-9}y^9.$$

Now, since we were asked to leave no negative exponents, we must apply the definition of $x^{-n} = \frac{1}{x^n}$ to get rid of the negative exponent in x^{-9}:

$$\begin{aligned}x^{-9}y^9 &= \frac{1}{x^9}y^9 \\ &= \frac{y^9}{x^9}.\end{aligned}$$

△

Roots

The next thing we consider is in a sense an "undoing" of exponents. Say, for example, we know that $x^3 = 8$ for some value x, and that we are interested in finding such an x. Well, one way is to guess and check (here $x = 2$ works since $2^3 = 8$). This idea is what motivates the following definitions.

Definition (Roots)**.** Suppose that for some positive integer n, $y^n = x$. Then we say that y is *an n***th root** of x.

We note that if, say, $y^2 = 4$, that both $y = 2$ and $y = -2$ are the second (or square) roots of 4 (note $2^2 = (-2)^2 = (-1)^2 2^2 = 4$). This is why we say y is **an** nth root, rather than necessarily saying y is **the** nth root (note, however, that sometimes there is a single nth root). So, for example, 4 has two square roots, but 27 has only one third root (namely 3 since $3^3 = 27$). Thus, we have the following notations for when $y^n = x$:

- **i.** If n is odd, we write $y = \sqrt[n]{x} = x^{1/n}$ and note that there is only one real root of x in this situation.
- **ii.** If n is even and x is negative, there are no real roots of x (the classic example is when $x^2 = -1$)
- **iii.** If n is even and x is positive, we write $y = \sqrt[n]{x} = x^{1/n}$ to mean the positive root of x.
- **iv.** If $n = 2$, we write $y = \sqrt{x}$.

Like the case with integer exponents, we have some nice rules that go with taking roots of numbers.

Theorem (Properties of Roots)**.** *Let n be a positive integer and assume x and y take on values that make sense. Then,*

- ***i.*** $\sqrt[n]{xy} = \sqrt[n]{x}\,\sqrt[n]{y} = (xy)^{1/n} = x^{1/n}y^{1/n}$
- ***ii.*** $\sqrt[n]{\frac{x}{y}} = \frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \left(\frac{x}{y}\right)^{1/n} = \frac{x^{1/n}}{y^{1/n}}$

iii. $\sqrt[m]{\sqrt[n]{x}} = \sqrt[mn]{x} = (x^{1/n})^{1/m} = x^{1/(mn)}$

Before we see an example, we note that we can combine the idea of positive integer exponents and roots to obtain what is called rational exponents: For example, the 5th root of x^2 of would be $x^{2/5}$. We define this formally as follows.

Definition (Rational Exponents)**.** Under the usual restrictions (avoiding 0^0, $1/0$, $\sqrt{-1}$, etc.), if m and n are integers, then

$$x^{m/n} = (x^{1/n})^m = (x^m)^{1/n} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}.$$

As above, these properties are mostly going to be used for expressions with variables, rather than just with numbers. So let's now see this applied to such an example.

Example 3. *Rewrite* $\sqrt[4]{\frac{2x^{16}}{y^{-7}}}$ *in the most simplified manner without using radicals or negative exponents.* We will first distribute the radical:

$$\begin{aligned} \sqrt[4]{\frac{2x^{16}}{y^{-7}}} &= \frac{\sqrt[4]{2x^{16}}}{\sqrt[4]{y^{-7}}} \\ &= \frac{\sqrt[4]{2}\,\sqrt[4]{x^{16}}}{\sqrt[4]{y^{-7}}}. \end{aligned}$$

Next, we rid ourselves of radicals:

$$\frac{\sqrt[4]{2}\,\sqrt[4]{x^{16}}}{\sqrt[4]{y^{-7}}} = \frac{2^{1/4}x^{16/4}}{y^{-7/4}}.$$

Lastly, we simplify our exponents (where we can) and rid ourselves of negative exponents

$$\frac{2^{1/4}x^{16/4}}{y^{-7/4}} = 2^{1/4}x^4y^{7/4}.$$

△

Polynomials

Perhaps the most fundamental function one should be aware of is the polynomial. Polynomials (and rational and exponential) functions will be the main focus in our calculus, and so we review polynomials first (rational functions will come next, and then exponentials later). Recall that given an expression, we may use letters instead of numbers as coefficients, and of course use letters as our variables; i.e., letters can be used for both constants and variables. For example, we can write expressions like $ax + b$, rather than something like $2x + 3$, with the understanding that a and b are fixed constant values. We use this to formally define a polynomial.

Definition (Polynomials)**.** Let n be a non-negative integer, and $a_1, a_2, \ldots, a_n$ real numbers with at least a_n non-zero. Then we call the expression

$$a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$$

an polynomial.

Note that we write $\ldots$ and $\cdots$ as an indication that the sequence of things goes on in the expected manner. For example, we could write $a_5, a_4, \ldots, a_1$ in place of a_5, a_4, a_3, a_2, a_1. Since the case for $n = 0, 1, 2, 3$ (with a_n non-zero) are so common, we give such polynomials names.

Definition. 1. Polynomials of the form a_0 are called constant polynomials.

2. Polynomials of the form $a_1x + a_0$, with $a_1 \neq 0$, are called linear polynomials.

3. Polynomials of the form $a_2x^2 + a_1x + a_0$, with $a_2 \neq 0$, are called quadratic polynomials.

4. Polynomials of the form $a_3x^3 + a_2x^2 + a_1x + a_0$, with $a_3 \neq 0$, are called cubic polynomials.

The polynomials $5, 3x + 2, 4x^2, 3x^3 + x$ are examples of constant, linear, quadratic, and cubic polynomials, respectively.

We note that polynomials behave like real numbers in that they have the same distributive, additive, subtractive, etc., properties as real numbers. We demonstrate this by example.

Example 4. *Simplify the following expression.*

1. $(ax^2 + bx + c) - (x^2 + dx + e)$
2. $(x - 1)(x + 1)$
3. $(2x + 2)(x^3 + 2x + 1)$

To do item one, we must distribute the negative sign to the rightmost polynomial, and then combine like-terms:

$$\begin{aligned}(ax^2 + bx + c) - (x^2 + dx + e) &= ax^2 + bx + c - x^2 - dz - e \\ &= (a - 1)x^2 + (b - d)x + c - e.\end{aligned}$$

To do item one, we do the so-called "FOIL" method, i.e., we distribute:

$$\begin{aligned}(x - 1)(x + 1) &= x(x + 1) - 1(x + 1) \\ &= x^2 + x - x - 1 \\ &= x^2 - 1.\end{aligned}$$

To do item three, we again distribute:

$$\begin{aligned}(2x + 2)(x^3 + 2x + 1) &= 2x(x^3 + 2x + 1) + 2(x^3 + 2x + 1) \\ &= 2x^4 + 4x^2 + 2x + 2x^3 + 4x + 2 \\ &= 2x^4 + 2x^3 + 4x^2 + 6x + 2.\end{aligned}$$

△

Factoring and solving polynomial equations

We do this section by example and the reminder of the quadratic formula.

Theorem. *Given the quadratic $p(x)$ above, the x such that $p(x) = 0$ is given by*

$$\begin{aligned}x_- &= \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ x_+ &= \frac{-b + \sqrt{b^2 - 4ac}}{2a}.\end{aligned}$$

Moreover, $p(x)$ can be factored into

$$p(x) = a(x - x_-)(x - x_+).$$

So let's see this applied to some polynomials.

Example 5. *Factor $x^2 + 2x + 1$.* If you are familiar with factoring techniques, you may be able to immediately factor this. In any case, we may apply the quadratic formula to $x^2 + 2x + 1 = 0$ to obtain

$$\begin{aligned}x &= \frac{-2 \pm \sqrt{2^2 - 4}}{2} \\ &= -1,\end{aligned}$$

where we have used $a = 1, b = 2, c = 1$. Note that the quadratic formula gives us *two* roots, namely, we have two roots being -1. Thus, we may factor $x^2 + 2x + 1$ to be $(x + 1)^2$. △

Example 6. *Factor* x^2+4x+3. We apply the quadratic formula to $x^2+4x+3=0$ with $a=1, b=4, c=3$:

$$\begin{aligned} x &= \frac{-4 \pm \sqrt{16-12}}{2} \\ &= \frac{-4 \pm \sqrt{4}}{2} \\ &= \frac{-4 \pm 2}{2} \\ &= -2 \pm 1; \end{aligned}$$

i.e., $x=-1$ and $x=-3$ are the solutions to $x^2+4x+3=0$ and so $(x+1)$ and $(x+3)$ are the factors of x^2+4x+3. Therefore, x^2+4x+3 is factored to $x^2+4x+3=(x+1)(x+3)$. △

Example 7. *Factor* x^3+2x^2+x. The trick here is to firstly factor out an x from the polynomial: $x^3+2x^2+x = x(x^2+2x+1)$. To see this, we needed to notice that *every* term in the polynomial had a factor of x. Next, we see that x^2+2x+1 was factored above to be $(x+1)^2$, and so our new polynomial factors to$x^3+2x^2+x = x(x+1)^2$. △

Example. *Factor* $2x^2+8x+6$ The trick here to to first factor out the coefficient of x^2: $2x^2+8x+6 = 2(x^2+4x+3)$. Then factor what is inside the parentheses. But x^2+4x+3 was factored above to be $(x+1)(x+3)$, and therefore $2x^2+8x+6 = 2(x+1)(x+3) = (2x+2)(x+3)$.

Another way to factor this is to use the quadratic formula. Indeed, we have

$$\begin{aligned} x_- &= \frac{-8 - \sqrt{8^2 - 4\cdot 6 \cdot 2}}{2\cdot 2} \\ &= -3 \\ x_+ &= \frac{-8 + \sqrt{8^2 - 4\cdot 6 \cdot 2}}{2\cdot 2} \\ &= -1, \end{aligned}$$

and so, by the theorem above, we can factor $2x^2+8x+6$ into

$$\begin{aligned} 2x^2+8x+6 &= 2(x-x_-)(x-x_+) \\ &= 2(x-(-3))(x-(-1)) \\ &= 2(x+3)(x+1) \\ &= (2x+2)(x+3). \end{aligned}$$

We see one last example that pertains to showing a solution doesn't exist to $p(x)=0$ for some polynomial p.

Example 8. *Determine if* $x^2-x+1=0$ *has a solution; i.e., determine if we can factor* x^2-x+1. To do so, we go straight to the quadratic formula as usual, with $a=c=1, b=-1$:

$$\begin{aligned} x &= \frac{1 \pm \sqrt{1-4}}{2} \\ &= \frac{1 \pm \sqrt{-3}}{2}, \end{aligned}$$

and so, since we are restricting ourselves to real values, $x^2-x+1=0$ does not have any real solutions. △

As the last example demonstrates, a shortcut to determining if $ax^2+bx+c=0$ has a solution, one just needs to check if $\sqrt{b^2-4ac}$ is defined; i.e., if $b^2-4ac \geq 0$. Let's state this as a theorem.

Theorem. *For a quadratic* ax^2+bx+c, *the equation* $ax^2+bx+c=0$ *has a real solution if and only if*

$$b^2 - 4ac \geq 0.$$

Rational Expressions

In a word, by a rational expression we mean an expression of quotients of polynomials. For example, the expressions

$$\frac{x+1}{x-1}$$

and

$$x^2 - \frac{x-1}{x^2+3} + \frac{x}{x^4+2}$$

are rational expression. It turns out that, like polynomials, we can treat rational expressions like we would real numbers, and in particular, they behave just like fractions. We demonstrate by examples.

Example 9. *Simplify the expression* $\frac{2}{x} + \frac{x}{x+1}$ *so that there is at most one fraction.*

We shall effectively pretend we are simplifying a expression containing fractions. Namely, we first make it so both fraction have a common denominator:

$$\begin{aligned} \frac{2}{x} + \frac{x}{x+1} &= \frac{2}{x}\frac{x+1}{x+1} + \frac{x}{x+1}\frac{x}{x} \\ &= \frac{2(x+1)}{x(x+1)} + \frac{x(x)}{(x+1)x} \\ &= \frac{2x+2}{x(x+1)} + \frac{x^2}{x(x+1)}. \end{aligned}$$

At this point we may now combine the two fractions and simplify completely,

$$\frac{2x+2}{x(x+1)} + \frac{x^2}{x(x+1)} = \frac{2x+2+x^2}{x(x+1)}.$$

△

Example 10. *Simplify the expression* $x + \frac{x+1}{x-1}$ *so that there is at most one fraction.*

We shall effectively pretend we are simplifying a expression containing fractions:

$$\begin{aligned} x + \frac{x+1}{x-1} &= \frac{x}{1} + \frac{x+1}{x-1} \\ &= \frac{x}{1}\frac{x-1}{x-1} + \frac{x+1}{x-1}\frac{1}{1} \\ &= \frac{x(x-1)}{(x-1)} + \frac{x+1}{x-1} \\ &= \frac{x(x-1)+x+1}{x-1} \\ &= \frac{x^2-x+x+1}{x-1} \\ &= \frac{x^2+1}{x-1}. \end{aligned}$$

△

Example 11. *Simplify the expression* $\frac{1}{\frac{x}{x+h}} - \frac{1}{2x}$. The tricky part here is dealing with $\frac{1}{\frac{x}{x+h}}$. But if we recall something like, say, $\frac{1}{\frac{1}{7}}$, we might remember that to deal with this, we bring the bottommost number to the numerator:

$$\frac{1}{\frac{1}{7}} = \frac{7}{1} = 7.$$

The same sort of thing applies to rational expression:

$$\frac{1}{\frac{x}{x+h}} = \frac{x+h}{x}.$$

Thus,

$$\begin{aligned}
\frac{1}{\frac{x}{x+h}} - \frac{1}{2x} &= \frac{x+h}{x} - \frac{1}{2x} \\
&= \frac{x+h}{x}\frac{2x}{2x} - \frac{1}{2x}\frac{x}{x} \\
&= \frac{2x(x+h)}{2x^2} - \frac{x}{2x^2} \\
&= \frac{2x(x+h)-x}{2x^2} \\
&= \frac{2x^2+2xh-x}{2x^2} \\
&= \frac{2x^2+x(2h-1)}{2x^2} \\
&= \frac{2x+2h-1}{2x}
\end{aligned}$$

△

Solving rational equations

In a previous section, we saw how to solve equations that looked like $ax^2 + bx + c = 0$. Here we shall see how to solve some equations that involve rational expression. We do so by example.

Example 12. *Solve for x in $\frac{1}{x} - 1 = 0$.* At first glance, the problem seems weird and impossible. But what happens if we were to multiply through by x? We would get

$$x\left(\frac{1}{x} - 1\right) = \frac{x}{x} - x = 1 - x = 0.$$

Thus, we see that, by solving for x in $1-x=0$, $x=1$ is a solution to the rational equation at hand. △

The next example may seem silly but it demonstrate a subtle point about solutions to rational equations.

Example 13. *Solve for x in $\frac{x(x+1)}{x} = 0$.* Yes, it is true that we can cancel out an x an get

$$\frac{x(x+1)}{x} = x + 1 = 0,$$

and so $x = -1$ is a solution to the equation. However, even though $x(x+1) = 0$ for both $x = -1$ *and* $x = 0$, $x = 0$ is not a solution to $\frac{x(x+1)}{x} = 0$ since we would then be dividing by zero. This may seem weird and arbitrary, but strictly speaking, it is mathematically sound, though perhaps a convention. △

To see where this sort of issue might arise, let us consider the next example.

Example 14. *Solve for x in $\frac{x-2}{x-1} + \frac{1}{(x-1)x}$.* We do our usual computations:

$$\begin{aligned}
\frac{x-2}{x-1} + \frac{1}{(x-1)x} &= \frac{x-2}{x-1}\frac{x}{x} + \frac{1}{(x-1)x} \\
&= \frac{x(x-2)}{x(x-1)} + \frac{1}{(x-1)x} \\
&= \frac{x(x-2)+1}{x(x-1)} \\
&= \frac{x^2-2x+1}{x(x-1)}.
\end{aligned}$$

At this point, we see we need to factor the polynomial in the numerator to solve for x:

$$\begin{aligned}\frac{x^2-2x+1}{x(x-1)} &= \frac{(x-1)^2}{x(x-1)} \\ &= \frac{(x-1)}{x}.\end{aligned}$$

So, $x = 1$ is a solution, right? This is wrong! If we were to input $x = 1$ into $\frac{x-2}{x-1} + \frac{1}{(x-1)x}$, we would end up dividing by zero! So, the point here is that after solving simplifying and solving as far as we can, we *must* make sure the values of x we obtain make sense with respect to the original equation. △

Inequalities

In this section we shall explore how to solve inequalities. This will become extremely important in future material and so understanding this section is extremely crucial. We start off with a reminder about some properties of inequalities.

Theorem (Properties of inequalities)**.** *Suppose $a > b$. Then*

1. $a + c > b + c$
2. *Multiplying the inequality by a positive number doesn't change the inequality: if $c > 0$, then $ac > bc$.*
3. *Multiplying the inequality by a negative number do change the inequality: if $c < 0$, then $ac < bc$.*

Now, when we say "solve an inequality," we mean to find all x such that the inequality is satisfied. For example, if we say solve the inequality $x + 3 > 0$, we would conclude all $x > -3$ would solve this inequality. Or say we want to solve $x + 3 < 2x + 2$, we would solve for x as we normally would in an equation:

$$x + 3 < 2x + 2 \text{ becomes } 3 < x + 2 \text{ by subtracting } x \text{ from both sides}$$

$$3 < x + 2 \text{ becomes } 1 < x \text{ by subtracting } 2 \text{ from both sides}$$

and so the values of x such that $x > 1$ solves the inequality $x+3 < 2x+2$. Before we see more examples, we recall that the absolute value function $|x|$ returns x if $x \geq 0$, and $-x$ if $x \leq 0$ (e.g., $|5| = |-5| = 5$). Then we have the following property

Theorem (Properties of absolute value inequalities)**.** *Given a positive number a, then $|x| < a$ means $-a < x < a$ and $|x| > a$ means $x < -a$ or $a < x$.*

We now go about solving some inequalities.

Example 15. *Solve the inequality* $2 < 3x + 3 < 6$. To do this problem, we manipulate the inequality until x is left by itself. Thus, we first subtract by 3 and then divide by three:

$$\begin{aligned}2 &< 3x + 3 < 6 \\ -1 &< 3x < 3 \\ -\frac{1}{3} &< x < 1.\end{aligned}$$

That is, x solves the inequality $2 < 3x + 3 < 6$ if and only if $-\frac{1}{3} < x < 1$. △

Example 16. *Solve the inequality* $x^2 + x < -2$. This problem is a bit trickier. We will need to obtain a polynomial, factor it, and then analyze where the factors are positive and negative. We first move the -2 to the left hand side to obtain:

$$x^2 + x - 2 < 0.$$

We then factor $x^2 + x - 2$:

$$x^2 + x - 2 = (x - 1)(x + 2).$$

Thus we wish to find for what x does the inequality $(x - 1)(x + 2) < 0$ hold.

We note that $x - 1 < 0$ when $x < 1$ and $x - 1 > 0$ when $x > 1$. Similarly, $x + 2 < 0$ when $x < -2$ and $x+2 > 0$ when $x > -2$. We can use this to find when $(x-1)(x+2) < 0$ by the following sign chart.

	$x < -2$	$-2 < x < 1$	$1 < x$
$(x-1)$	$---$	$---$	$+++$
$(x+2)$	$---$	$+++$	$+++$
$(x-1)(x+2)$	$+++$	$---$	$+++$

Therefore, we conclude that only $-2 < x < 1$ solves the inequality $(x-1)(x+2) < 0$ and thus the inequality $x^2 + x < -2$. △

Example 17. *Solve the inequality* $|x+2| < 2$. Here, we apply the theorem above about absolute value inequality:

$$|x+2| < 2 \text{ if and only if } -2 < x+2 < 2.$$

Thus, by subtracting 2 from both sides, we obtain $-4 < x < 0$ solves the inequality $|x+2| < 2$. △

Example 18. *Solve the inequality* $\frac{x+2}{x^2-1} < 0$ The problem is similar to the one with the quadratic in that we shall need to analyze where certain factors are positive or negative. So, firstly, we factor the denominator:

$$\frac{x+2}{x^2-1} = \frac{x+2}{(x-1)(x+1)}.$$

That is, we have transplanted the problem to the one where we must solve the inequality

$$\frac{x+2}{(x-1)(x+1)} < 0.$$

Thus, we need to determine where the factors $x+2$, $x-1$, and $x+1$ are respectively positive or negative. We find that $x+2 < 0$ when $x < -2$, and $x+2 > 0$ when $x > -2$. That $x-1 < 0$ when $x < 1$, and $x-1 > 0$ when $x > 1$. And that $x+1 < 0$ when $x < -1$, and $x+1 > 0$ when $x > -1$. We determine when

$$\frac{x+2}{(x-1)(x+1)} < 0.$$

by the following sign chart.

	$x < -2$	$-2 < x < -1$	$-1 < x < 1$	$1 < x$
$x+2$	$---$	$+++$	$+++$	$+++$
$x-1$	$---$	$---$	$---$	$+++$
$x+1$	$---$	$---$	$+++$	$+++$
$\frac{x+2}{(x-1)(x+1)}$	$---$	$+++$	$---$	$+++$

and so if x is such that $x < -2$ or $-1 < x < 1$, then x solves the inequality

$$\frac{x+2}{(x-1)(x+1)} < 0.$$

△

Remark. Notice that when using the sign charts, to find the desired intervals, we use where the expression equaled zero or was undefined to find the breakup points.