

Directions: Complete the following problems in the space below or on the back of the paper. To receive any credit, you must show your work.

Problem 1. Solve for x :

$$2 \log_8 \sqrt{x} + \log_8(x + 2) = 0.$$

Solution.

$$\begin{aligned} 2 \log_8 \sqrt{x} + \log_8(x + 2) &= \log_8(\sqrt{x})^2 + \log_8(x + 2) \\ &= \log_8 x + \log_8(x + 2) \\ &= \log_8 x(x + 2) \\ &= 0, \end{aligned}$$

and so by exponentiating by 8, we get

$$x(x + 2) = 8^{\log_8 x(x+2)} = 8^0 = 1$$

and so we solve $x(x + 2) = 1$, which is equivalent to solving $x^2 + 2x - 1 = 0$. Using the quadratic formula, we conclude the probable solutions are

$$x = -1 \pm \sqrt{2}.$$

We now have to check whether or not these solutions make sense with respect to the original equation. We find

$$-1 - \sqrt{2} < 0$$

and so $1 - \sqrt{2}$ is not a solution. But $-1 + \sqrt{2} > 0$ and $-1 + \sqrt{2} + 2 > 0$ and so $-1 + \sqrt{2}$ is a solution.

Problem 2. Second National Bank offers an account that earns 4.42% per year, compounded continuously. If a person invests \$11,000 in this account, what will be the value of the account at the end of 12 years? (Round your answer to the nearest cent.)

Solution. Here $P = 11,000$, $r = 4.42\% = 0.042$, $t = 12$, and $F = Pe^{rt}$ since we are compounding continuously. Thus $F = 11,000e^{0.042 \cdot 12} \approx 18,208.62$.

Problem 3. Suppose we are given the revenue and cost models

$$\begin{aligned} R(x) &= -2x^2 + 2x + 16 \\ C(x) &= 3x - 5 \end{aligned}$$

(a) Find the profit function.

(b) Find the break even quantity (take note of the signs of your possible answers).

Solution. The profit function is given by $P(x) = R(x) - C(x) = -2x^2 + 2x + 16 - (3x - 5) = -2x^2 - x + 21$. Using the quadratic formula, we get that the break even quantity is $x = 3$ or $x = -\frac{7}{2}$, but a quantity of $-\frac{7}{2}$ doesn't make sense. So the answer is $x = 3$.