More Review

In this week, we review how to construct lines given some points, and then review the basics about functions.

Line

Here we will briefly go over finding slopes of lines, point slop form, finding equations for lines vertical, horizontal and otherwise, and the slope-intercept form.

Definition 1. Let L be a line with two distinct points (x_1, y_1) and (x_2, y_2) that lie on it. If $x_1 \neq x_2$, then we define the slope m of L to be

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

Note that if, say, the $m > 0$, then m tells you how fast the line is increasing: if m is small, then the line is increasing very slowly, and if m is large, the line is increasing very quickly.

It turns out that given just two points on a line, or just one point and the slope, one may find an equation that describes the line.

Theorem (Point-Slope Form)**.** *Let L be a line. If its slope m is given and it is known that (x_1, y_1) lies on L, then*

$$y - y_1 = m(x - x_1)$$

is an equation that describes the line.

If m is unknown, but it is known that both (x_1, y_1) and (x_2, y_2) lie on the line, the equation of the line can be found by first finding the slope and then applying the result above.

Next, if the slope is given our found, and the y-intercept b is known, then one has immediately

$$y = mx + b$$

is an equation that describes the given line.

Example 1. *Find the equation for the line passing through the points $(0, 1)$ and $(4, 9)$.*

We shall first find the slope m of the given line and then use the point slope form. First,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{9 - 1}{4 - 0} = \frac{8}{4} = 2,$$

where we have used $x_1 = 0$,$x_2 = 4$,$y_1 = 1$,$y_2 = 9$. Thus, we choose at random the point $(0, 1)$ ($(4, 9)$ works just as well), use $m = 2$, and apply the point slope form:

$$y - 1 = 2(x - 0) = 2x.$$

is an equation for the given line. One may wish to solve for y, and in this case,

$$y = 2x + 1$$

is also an equation of the given line. △

Example 2. *Find the equation for the line passing through the points $(2, 3)$ and $(-2, 4)$.*

We find m first. Set $(x_1, y_1 = (2, 3)$ and $(x_2, y_2) = (-2, 4)$. Then

$$m = \frac{4 - 3}{-2 - 2} = -\frac{1}{4}.$$

Thus, by the point slope form,

$$y = m(x - x_1) + y_1 = -\frac{1}{4}(x - 2) + 3 = -\frac{1}{4}x + \frac{7}{2}$$

is the desired line. △

We end with two more definitions of specific types of lines and some examples.

Definition (Vertical Lines)**.** The equation $x = a$ for some value a describes a vertical line passing through any point of the form (a, y) for y a real number.

Definition (Horizontal Lines)**.** The equation $y = a$ for some value a describes a horizontal line passing through any point of the form (x, a) for x a real number.

Example 3. *Find the equation for a vertical and for a horizontal line that passes through the point* $(2, 3)$.

To solve this problem, we need only apply the above definitions. By the definition of a vertical line, we note that $x = 2$ works, since this equation describes a vertical line passing through any point of the form $(2, y)$, and so, in particular, the point $(2, 3)$. Similarly, by the definition of a horizontal line, $y = 3$ works for similar reasons. △