Assignment 1

Due: 1/25/18

All work must be shown in your solutions, even if it is not explicitly asked for you to explain. Use your own paper to turn in solutions.

Problem 1

Consider the following ODE

$$\left(\frac{dy}{dx}\right)^2 - e^{-2y} = 0.$$

(a) What is the order of the ODE. Explain in a single sentence.

(b) Is the ODE linear? Explain in a single sentence.

(c) What is the largest interval I the function

$$y(x) = \ln(x+1)$$

is a solution to the ODE? (*Hint: where are* $y = \ln(x+1)$ *and* y' *defined?*) Also, verify by direct computation that $y = \ln(x+1)$ is indeed a solution to the given ODE.

Problem 2

Let t =time, and $x(t)$ =displacement of a particle at time t. Suppose the particle satisfies the following IVP:

$$\begin{cases} \dfrac{d^2x(t)}{dt^2} = -\frac{\pi^2}{4}x(t) \\ x(1) = 1 \\ x(2) = 2 \end{cases}.$$

It can be shown (you don't need to show this) that $x(t)$ is of the form

$$x(t) = c_1 \cos\left(\frac{\pi}{2}t\right) + c_2 \sin\left(\frac{\pi}{2}t\right),$$

where c_1, c_2 are two constants.

(a) Suppose that we wish for the particle to have a displacement of -1 at time $t = 3$ (i.e., $x(3) = -1$). Under the given IVP, does this happen? *Hint: Find* c_1, c_2 *based off of the initial conditions. You will then have a particular solution* $x(t)$*. Using this particular solution, you may test whether or not the particle attains a displacement of* -1 *at time* $t = 3$*.*

Now suppose the particle satisfies the following IVP:

$$\begin{cases} \dfrac{d^2x(t)}{dt^2} = -\dfrac{\pi^2}{4}x(t) \\ x(1) = ? \\ x(2) = 2 \end{cases},$$

where $x(1)$ is to be determined. Again, $x(t)$ takes the form as above.

(b) Suppose now that we wish for the particle to have a displacement of -2 at time $t = 3$. What value for $x(1)$ guarantees this will happen? *Hint: This problem translates to solving the IVP with initial conditions* $x(2) = 2, x(3) = -2$*, and then computing* $x(1)$*.*