Note: The following is not necessarily an exhaustive list of the material you will need to know, nor the type of problems you need to know. This is not a legal binding document of what I have to put on the exam, or what I cannot put on the exam.

Topics

Chapter 1

section 1.1

1. What is an ODE?
2. How do you find the order of an ODE?
3. How do you distinguish between linear and non-linear ODEs?
4. What is an explicit solution?
5. What is an implicit solution?
6. What does it mean to verify a function is a solution?

section 1.2

1. What is an IVP?
2. Given a 1 or 2 parameter family of solutions to an ODE, can you solve for the parameters given sufficiently many initial conditions?
3. What is the existence and uniqueness theorem, and what are its implications?
4. How do you use the existence and uniqueness theorem? E.g., if I tell you I have two distinct solutions to $y' = xy$ with initial condition $y(0) = 0$, why do you know I'm lying?
5. What does it mean to verify a function is a solution to an IVP?

section 1.3

1. How do you translate from English to an IVP or ODE? E.g., problems given in WebAssign and in class.
2. If I describe the rate of change of a process (i.e., some function), can you translate that into a statement about derivatives?

Chapter 2

section 2.1

1. Given a direction field, can you sketch a solution satisfying a specified initial condition?

section 2.2

Let $y' = f(x, y)$ be a separable ODE.

1. Can you apply a separation of variables?
2. Can you solve it via a separation of variables?
3. Can you find the constant solutions?

section 2.3

Let $a_1(x)y' + a_0(x)y = f(x)$ be a linear ODE.

1. Can you find the integrating factor $\mu(x)$?
2. Can you find the general solution to the ODE using integrating factors?
3. Can you determine the transient terms?
4. Can you find the singular points of the ODE?

section 2.5

1. Given a function, can you test whether it is homogeneous?
2. Given a ODE, can you test if it homogeneous?
3. Given a homogeneous ODE, can you solve it by substitution?
4. Given a Bernoulli's equation, can you place it into a linear ODE?
5. Given a Bernoulli's equation, can you solve it by substitution?
6. Given an ODE $y' = f(Ax + By + C)$, can you solve it by a substitution?

section 2.6 Consider the IVP:

$$\begin{cases} y' = f(x, y) \\ y(x_0) = y_0 \end{cases}.$$

1. Using Euler's method, can you sketch a solution curve?
2. Using Euler's method, can you approximate $y(x_0 + nh)$ for some h? I.e., can you run Euler's method n times to approximate an evaluation of the solution to the IVP?

Misc. Material

1. Proficiency in algebra is expected. If you are rusty with any algebra, you should review it. While dropping a negative may not hurt your score much or possibly at all, not being able to simplify $e^{-\ln x}$, or finding domains of basic functions might.
2. Relevant "hard" integrals will be provided. E.g., $\int \tan$ may be provided, but $\int \cos$ will not.
3. Finding the largest interval of definition for any of the above sections may be asked.

Problems

The following problems are to give you a feel for what you might expect for the exam. The exam will be longer than three questions.

Problem 1

Consider the following ODE

$$xy' = \tan(y).$$

(a) Is it linear or nonlinear? What is its order?

(b) Does this ODE have the trivial solution?

(c) Solve the ODE using a separation of variables?
Hint: $\int \cot(x)dx = \log(\sin(x)) + c$.

Problem 2

Verify $y = \sin(x) - \cos(x) + 2$ is a solution to the IVP

$$\begin{cases} y'' + y = 2 \\ y(0) = 1 \\ y'(0) = 1. \end{cases}$$

Why can we apply the existence and uniqueness theorem to conclude that y is the only solution to this IVP.

Problem 3

Solve the following ODE by using substitution to reduce it to a separation of variables:

$$y' = \sqrt{x + y + 1} + x + y, \quad x > 0.$$

Hint: $\int \frac{1}{\sqrt{x}+x} dx = 2\ln(\sqrt{x} + 1) + c$. (Solving this integral is actually pretty neat. Granted my wife solved it because I didn't want to, but her solution was neat.)