# Assignment 5

Due: 3/22/18

*All relevant work must be shown in your solutions, even if it is not explicitly asked for you to explain.*

The two problems are from the book.

## Problem 1

(3pts)

Suppose

$$X_1(t) = \begin{pmatrix} 1 \\ 6 \\ -13 \end{pmatrix}, \quad X_2(t) = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix} e^{-4t}, \quad X_3(t) = \begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} e^{3t}$$

are solutions to some system $X'(t) = A(t)X(t)$, where $A(t)$ is an $3 \times 3$ matrix, and $X(t)$ is a $3 \times 1$ vector. Determine whether or not the set of solutions $X_1, X_2, X_3$ form a fundamental set of solutions on $I = \mathbb{R}$. Hint: use Theorem 8.1.3 and Definition 8.1.3.

*Warning*: Being linearly independent is not enough. You need to take note of the dimensions.

## Problem 2

(3pts)

Prove that the general solution of the homogeneous linear system

$$X'(t) = \begin{pmatrix} 0 & 6 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} X(t)$$

on the interval $(-\infty, \infty)$ is

$$X(t) = c_1 \begin{pmatrix} 6 \\ -1 \\ -5 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} e^{3t}$$

Hint: use Theorem 8.1.5 from the text. In particular, identify appropriate $X_1, X_2, X_3$, show they form a fundamental set of solutions, and then apply the theorem.

*Warning*: each $X_1, X_2, X_3$ needs to be shown to be a solution. I will accept you only showing one is a solution so long as you mention the others can be checked similarly.