The Laplace Transform

Properties

Laplace Transform:	$F(s) = \mathscr{L}\{f(t)\} = \int_0^\infty e^{-st} f(t)dt$
Linearity:	$\mathscr{L}\{\alpha f(t) + \beta g(t)\} = \alpha\mathscr{L}\{f(t)\} + \beta\mathscr{L}\{g(t)\}$
	$\mathscr{L}^{-1}\{\alpha F(s) + \beta G(s)\} = \alpha\mathscr{L}^{-1}\{F(s)\} + \beta\mathscr{L}^{-1}\{G(s)\}$
Laplace of Derivatives:	$\mathscr{L}\{f'(t)\} = sF(s) - f(0)$
	$\mathscr{L}\{f''(t)\} = s^2F(s) - sf(0) - f'(0)$
	$\mathscr{L}\left\{f^{(n)}(t)\right\} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0)$
Inverse Laplace of Derivatives:	$\mathscr{L}\{tf(t)\} = -\frac{d}{ds}\mathscr{L}\{f(t)\}$
	$\mathscr{L}\left\{t^2f(t)\right\} = \frac{d^2}{ds^2}\mathscr{L}\{f(t)\}$
	$\mathscr{L}\{t^nf(t)\} = (-1)^n\frac{d^n}{ds^n}F(s)$
Translation in s:	$\mathscr{L}\{e^{at}f(t)\} = F(s-a)$
	$\mathscr{L}^{-1}\{F(s-a)\} = e^{at}f(t)$
Translation in t:	$\mathscr{L}\{f(t-a)\mathscr{U}(t-a)\} = e^{-as}F(s)$
	$\mathscr{L}^{-1}\{e^{-as}F(s)\} = f(t-a)\mathscr{U}(t-a)$
	$\mathscr{L}\{g(t)\mathscr{U}(t-a)\} = e^{-as}\mathscr{L}\{g(t+a)\}$
Convolution:	$f * g = \int_0^t f(\tau)g(t-\tau)d\tau$
Laplace of Convolution:	$\mathscr{L}\{f * g\} = \mathscr{L}\{f(t)\}\mathscr{L}\{g(t)\} = F(s)G(s)$
Inverse Laplace of Product:	$\mathscr{L}^{-1}\{F(s)G(s)\} = f * g$
Laplace of Integral:	$\mathscr{L}\left\{\int_0^t f(\tau)d\tau\right\} = \frac{F(s)}{s}$
Laplace of T-Periodic Function:	$\mathscr{L}\{f(t)\} = \frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)dt$

Heaviside Function

Heaviside Function: $\mathscr{U}(t-a) = \begin{cases} 0, & 0 \le t < a \\ 1, & t \ge a \end{cases}$

$$f(t) = \begin{cases} g(t), & 0 \le t < a \\ h(t), & t \ge a \end{cases} \iff f(t) = g(t) - g(t)\mathscr{U}(t-a) + h(t)\mathscr{U}(t-a)$$

$$f(t) = \begin{cases} 0, & 0 \le t < a \\ g(t), & a \le t < b \\ 0, & t \ge b \end{cases} \iff f(t) = g(t)[\mathscr{U}(t-a) - \mathscr{U}(t-b)]$$

$$\mathscr{L}\{\mathscr{U}(t-a)\} = \frac{e^{-as}}{s}$$

Common Laplace Transforms

$$\mathscr{L}\{1\} = \frac{1}{s} \qquad \mathscr{L}\{t^n\} = \frac{n!}{s^{n+1}} \qquad \mathscr{L}\{e^{at}\} = \frac{1}{s-a}$$

$$\mathscr{L}\{\cos kt\} = \frac{s}{s^2+k^2} \qquad \mathscr{L}\{\sin kt\} = \frac{k}{s^2+k^2}$$

$$\mathscr{L}\{\sinh kt\} = \frac{k}{s^2-k^2} \qquad \mathscr{L}\{\cosh kt\} = \frac{s}{s^2-k^2}$$

$$1 = \mathscr{L}^{-1}\left\{\frac{1}{s}\right\} \qquad t^n = \mathscr{L}^{-1}\left\{\frac{n!}{s^{n+1}}\right\} \qquad e^{at} = \mathscr{L}^{-1}\left\{\frac{1}{s-a}\right\}$$

$$\sin kt = \mathscr{L}^{-1}\left\{\frac{k}{s^2+k^2}\right\} \qquad \cos kt = \mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\}$$

$$\sinh kt = \mathscr{L}^{-1}\left\{\frac{k}{s^2-k^2}\right\} \qquad \cosh kt = \mathscr{L}^{-1}\left\{\frac{s}{s^2-k^2}\right\}$$

$$\mathscr{L}\{\mathscr{U}(t-a)\} = \frac{e^{-as}}{s}$$