3.4 Local Linearity

- Linear Approximation

Quiz on this Section:

- Friday 3/15/19

WebAssign on this section:

- Sunday 3/10/19

Warm Up:

- Let $f(x) = 2x^2 - 12x$. Find the x-value(s) where the graph of $f(x)$ has a horizontal tangent line.
- Let $g(x) = 2e^x(x^2 - 3x)$. Find $g'(x)$.

Warm up (cont.)

Linear Approximation

- Approximating the Change in y

> If $y = f(x)$ is differentiable at $x = c$, then the change in y, given by $f(c+h) - f(c)$ can be approximated by the linear function $f'(c)h$. That is,
> $$f(c+h) - f(c) \approx f'(c)h$$
> if h is small.

- Tangent Line Approximation

> If $y = f(x)$ is differentiable at $x = c$, then for values of x near c,
> $$f(x) \approx f(c) + f'(c)(x - c)$$
> Thus, for values of x near c, the graph of the curve $y = f(x)$ is approximately the same as the graph of the tangent line through the point $(c, f(c))$.

Find the linear approximation of the function $f(x) = \sqrt{1-x}$ at $a = 0$. Use $L(x)$ to approximate the numbers $\sqrt{0.9}$ and $\sqrt{0.99}$. (Round your answers to four decimal places). Illustrate by graphing f and its tangent line

Want to compute $L(x) = f'(a)(x-a) + f(a)$ with $a = 0$.

$f'(x) = -\frac{1}{2\sqrt{x}}$, so $f'(0) = -\frac{1}{2}$.

$f(0) = 1$.

So $L(x) = -\frac{1}{2}x + 1$.

Note $\sqrt{0.9} = f(0.1) \cong L(0.1) = 0.95$

Note $\sqrt{0.99} = f(0.01) \cong L(0.01) = 0.995$.

Assume that it costs a company approximately

$$C(x) = 400,000 + 120x + 0.002x^2$$

dollars to manufacture x smartphones in an hour.

(a) Find the marginal cost function. Use it to estimate how fast the cost is increasing when $x = 10{,}000$. Compare this with the exact cost of producing the 10,001st smartphone.

Marginal cost: $C'(x)$. Compute $C'(10,000) = 160$, so the cost is increasing at a rate of 160 per smartphone. The exact cost of producing the 10,001st smartphone is $C(10,001) - C(10,000) = 160.002$. Thus, there is a difference of 0.002.

(b) Find the average cost function $\bar{C}$ and the average cost to produce the first 10,000 smartphones. Average cost is $\bar{C}(x) = \frac{C(x)}{x}$, and so

$$\bar{C}(10,000) = \frac{C(10,000)}{10,000} = 180.$$

(c) Using your answers to parts (a) and (b), determine whether the average cost is rising or falling at a production level of 10,000 smartphones.

The marginal cost from (a) is lower than the average cost from (b). This means that the average cost is falling at a production level of 10,000 smartphones.

Suppose $P(x)$ represents profit on the sale of x Blu-ray discs. If $P(1,000) = 9,000$ and $P'(1,000) = -1$, what do these values tell you about the profit?

$P(1,000)$ represents the profit on the sale of 1000 Blu-ray discs. $P(1,000) = 9,000$, so the profit on the sale of 1000 Blu-ray discs is 9000.

$P'(x)$ represents the rate of change of the profit as a function of x. $P'(1,000) = -1$, so the profit is decreasing at the rate of 1 per additional Blu-ray disc sold.

The Audubon Society at Enormous State University (ESU) is planning its annual fund-raising "Eatathon." The society will charge students \$1.10 per serving of pasta. The society estimates that the total cost of producing x servings of pasta at the event will be

$$C(x) = 350 + 0.10x + 0.002x^2 \text{ dollars.}$$

(a) Calculate the marginal revenue $R'(x)$ and profit $P'(x)$ functions.
Marginal revenue: $R'(x)$
Marginal Profit: $P'(x)$.
Compute: $R'(x) = 1.10$ and $P'(x) = 1 - 0.004x$.

(b) Compute the revenue and profit, and also the marginal revenue and profit, if you have produced and sold 200 servings of pasta. Interpret the results.

Revenue: 220

Profit: -230

Marginal Revenue: 1.10 per additional plate

Marginal Profit: 0.20 per additional plate

The approximate profit from the sale of the 201st plate of pasta is 0.20.

(c) For which value of x is the marginal profit zero? Interpret your answer.

$x = 250$ plates of pasta.

The graph of the profit function is a parabola with a vertex at x = 250, so the loss is at a minimum when you produce and sell 250 plates.