

1. Sketch the curve of the given function by answering the following parts:

$$f(x) = \frac{1}{x^2 - 2x}$$

- (a) Determine the domain of  $f(x)$  and determine, if they exist, any vertical and horizontal asymptotes.

$$D: (-\infty, 0) \cup (0, 2) \cup (2, \infty)$$

$$V.A.: x = 0, x = 2$$

$$H.A.: y = 0 \text{ both } -\infty / \infty.$$

- (b) Use the first derivative to find intervals on which  $f$  is increasing, respectively decreasing.

$$f'(x) = \frac{-2x + 2}{(x^2 - 2x)^2}$$

$$\left. \begin{array}{l} f(x) \text{ inc. on } (-\infty, 0) \cup (0, 1) \\ f(x) \text{ dec. on } (1, 2) \cup (2, \infty) \end{array} \right\} \begin{array}{l} x = 1 \text{ is} \\ \text{a rel. max} \end{array}$$

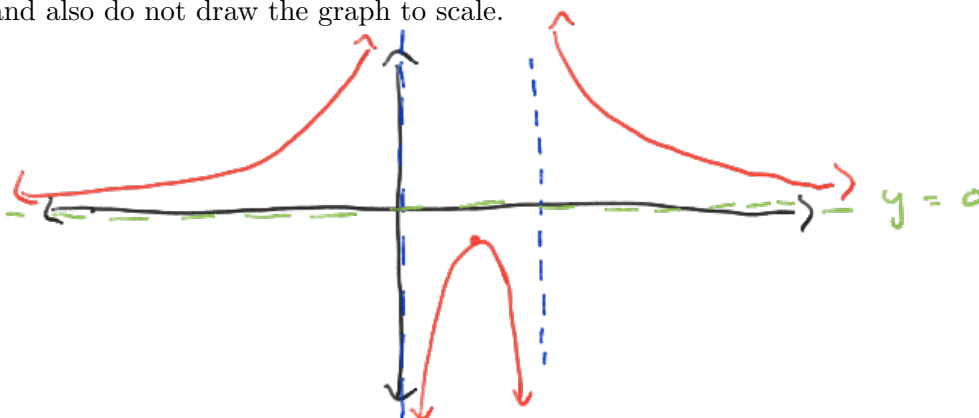
- (c) Use the second derivative to find the intervals of concavity.

$$f''(x) = \frac{-2x^2 + 4x + 2(2x - 2)^2}{(x^2 - 2x)^3} \quad \left. \vphantom{f''(x)} \right\} f'' \neq 0$$

$$f(x) \text{ is CCU on } (-\infty, 0) \cup (2, \infty)$$

$$\text{is CCD on } (0, 2)$$

- (d) Use the preceding parts to sketch graph of  $f(x)$ . Clearly indicate all important points and details and also do not draw the graph to scale.



2. When analysing  $f(x)$  we determined the following facts. Use these to sketch a graph of  $f(x)$ .

- $\lim_{x \rightarrow \infty} f(x) = 3$  and  $\lim_{x \rightarrow -\infty} f(x) = -1$ .
- $f(x)$  has vertical asymptotes at both  $x = -4$  and  $x = 2$ .
- $f'(x)$  is negative on  $(-\infty, -4) \cup (0, 2) \cup (2, \infty)$ , is positive on  $(-4, 0)$ , and is undefined at  $x = 0$ .
- $f''(x)$  is negative on  $(-\infty, -4) \cup (-4, -2) \cup (0, 2)$  and is positive on  $(-2, 0) \cup (2, \infty)$ .
- $f(x)$  passes through  $(-2, 0)$ ,  $(0, 4)$ , and  $(1, 0)$ .

include  
concavity

