Section 6.1: Antiderivatives

Idea: Undo differentiation.

Example

1) $x^2 \longrightarrow 2x$
   $x^2+1 \longrightarrow 2x$
   $x^2-3 \longrightarrow 2x$
   etc.

So if we know $f'(x) = 2x$, then we can at best say

$$f(x) = x^2 + C.$$

Definite integrals:

If $g(x) = f'(x)$, then write

$$\int g(x)\,dx = f(x) + C.$$

"$\int$ undoes $\frac{d}{dx}$"

f is called the most general antiderivative of g.

Rules:

1) $\int x^p\,dx = \frac{1}{p+1} x^{p+1} + C$ for $p \neq -1$ Note $\frac{d}{dx}\left(\frac{1}{p+1} x^{p+1} + C\right) = \frac{p+1}{p+1} x^p + 0 = x^p$

2) $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln|x| + C$ ← note absolute values

3) $\int e^x\,dx = e^x + C$

4) $\int a f(x)\,dx = a\int f(x)\,dx$, $\int f(x) + g(x)\,dx = \int f(x)\,dx + \int g(x)\,dx$

Warning: For Indefinite integrals: Never Forget the $+C$

Examples

$$\int x^2\,dx = \frac{1}{2+1} x^{2+1} + C = \frac{1}{3}x^3 + C$$

$$\int x^{-0.1}\,dx = \frac{1}{-0.1+1} x^{-0.1+1} + C = \frac{1}{0.9} x^{0.9} + C$$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1} + C = \frac{1}{\frac{3}{2}} x^{\frac{3}{2}} + C = \frac{2}{3} x^{\frac{3}{2}} + C$$

$$\begin{aligned}\int (1+x)(1-3x)\,dx &= \int 1 - 3x + x - 3x^2\,dx \\ &= \int 1 - 2x - 3x^2\,dx \\ &= x - x^2 - x^3 + C\end{aligned}$$

$$\int \frac{2+u}{u}\,du = \int \frac{2}{u} + 1\,du = 2\ln|u| + u + C.$$