

Section 6.2 Substitution

u-substitution:

Examples

1) $\int (x+1)^7 dx$:

$$u = x+1$$

$$\frac{du}{dx} = 1 \rightarrow dx = du$$

$$\hookrightarrow \int (x+1)^7 dx = \int u^7 du = \frac{1}{8} u^8 + C = \frac{1}{8} (x+1)^8 + C$$

2) $\int x e^{x^2+1} dx$

$$u = x^2+1$$

$$\frac{du}{dx} = 2x \rightarrow dx = \frac{du}{2x}$$

$$\hookrightarrow \int x e^{x^2+1} dx = \int x e^u \frac{du}{2x} = \frac{1}{2} \int e^u du = \frac{1}{2} e^u + C = \frac{1}{2} e^{x^2+1} + C$$

3) $\int \frac{2}{(1-7t)^5} dt$

$$u = 1-7t$$

$$\frac{du}{dt} = -7 \rightarrow dt = -\frac{1}{7} du$$

$$\begin{aligned} \hookrightarrow \int \frac{2}{(1-7t)^5} dt &= \int \frac{2}{u^5} \left(-\frac{1}{7}\right) du = -\frac{2}{7} \int u^{-5} du = \left(-\frac{2}{7}\right) \left(-\frac{1}{4}\right) u^{-4} + C \\ &= \frac{1}{14} u^{-4} + C \\ &= \frac{1}{14} (1-7t)^{-4} + C \end{aligned}$$

4) $\int (x^4+2)(10x+x^5+1)^3 dx$

$$u = 10x+x^5+1$$

$$\frac{du}{dx} = 10+5x^4 = 5(x^4+2) \rightarrow dx = \frac{du}{5(x^4+2)}$$

$$\begin{aligned} \int (x^4+2)(10x+x^5+1)^3 dx &= \int \cancel{(x^4+2)} u^3 \frac{du}{5\cancel{(x^4+2)}} \\ &= \frac{1}{5} \int u^3 du \\ &= \frac{1}{5} \frac{1}{4} u^4 + C \\ &= \frac{1}{20} (10x+x^5+1)^4 + C \end{aligned}$$