

## Section 6.5 The Fundamental Theorem of Calculus

Theorem (FTOC): If  $F(x) = \int f(x)$ , then

$$\int_a^b f(x) dx = F(x) \Big|_a^b = F(b) - F(a).$$

Example

1) Compute  $\int_4^{81} 18\sqrt{x} dx$ .

1. Find antiderivative:  $\int 18\sqrt{x} dx = \int 18x^{1/2} dx = \frac{18}{3/2} \cdot x^{3/2} + C = 12x^{3/2} + C.$

2. Set  $F(x) = 12x^{3/2}$  ( $\therefore$ , let  $C=0$ )

3. Use FTOC:

$$\int_4^{81} 18\sqrt{x} dx = 12x^{3/2} \Big|_4^{81} = 12(81)^{3/2} - 12(4)^{3/2} = 8652$$

2) Compute  $\int_5^B (8x - 9x^2) dx$ :

1.  $\int 8x - 9x^2 dx = 4x^2 - 3x^3 + C$

2.  $F(x) = 4x^2 - 3x^3$

3. Use FTOC:

$$\begin{aligned} \int_5^B 8x - 9x^2 dx &= 4x^2 - 3x^3 \Big|_5^B = 4B^2 - 3B^3 - (4 \cdot 5^2 - 3 \cdot 5^3) \\ &= 4B^2 - 3B^3 + 275 \end{aligned}$$

3) Compute  $\int_2^B (2x-3)^4 dx$

1.  $\int (2x-3)^4 dx = \frac{1}{2} \int u^4 du = \frac{1}{10} u^5 + C = \frac{1}{10} (2x-3)^5 + C$

$$u = 2x - 3$$

$$du = 2 dx$$

$$dx = \frac{1}{2} du$$

2.  $F(x) = \frac{1}{10} (2x-3)^5$

3.  $\int_2^B (2x-3)^4 dx = \frac{1}{10} (2x-3)^5 \Big|_2^B = \frac{1}{10} (2B-3)^5 - \frac{1}{10} (2 \cdot 2 - 3)^5$   
 $= \frac{1}{10} (2B-3)^5 - \frac{1}{10}.$