Math 1071Q Integration Worksheet

Name:

Power Rule

The power rule for indefinite integrals is

$$\int x^p dx = \frac{1}{p+1}x^{p+1} + C, \quad p \neq -1.$$

Problem 1. For the following integrals, fill in the blank for the power rule.

1. $$\int x^7 dx = \frac{1}{\underline{7}+1}x^{\underline{7}+1} + \underline{C}$$

2. $$\int \frac{2}{x^2}dx = \frac{\underline{2}}{\underline{-2}+1}x^{\underline{-2}+1} + \underline{C}$$

3. $$\int x^{\underline{7}}dx = \frac{1}{8}x^8 + \underline{C}$$

Other Rules

The other integration rules are

$$\int x^{-1}dx = \ln|x| + C$$

$$\int e^x dx = e^x + C$$

$$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

$$\int af(x)dx = a\int f(x)dx$$

Problem 2. Use these rules to compute the following integrals.

1. $$\int x^{-2} - \frac{3}{x}dx = \frac{1}{-2+1}x^{-2+1} - 3\ln|x| + C = -x^{-1} - 3\ln|x| + C$$

2. $$\int -e^x - x^{-0.5}dx = -e^x - \frac{1}{-0.5+1}x^{-0.5+1} + C = -e^x - 2x^{1/2} + C$$

3. $$\int x(x^5 - 3)dx = \int x^6 - 3x\,dx = \frac{1}{6+1}x^{6+1} - \frac{3}{1+1}x^{1+1} + C = \frac{1}{7}x^7 - \frac{3}{2}x^2 + C$$

U-Substitution

An example of u-substitution is given:

To compute

$$\int xe^{2x^2+1}dx,$$

set $u = 2x^2 + 1$. Differentiating with respect to x gives $\frac{du}{dx} = 4x$. Solving for dx gives: $dx = \frac{1}{4x}du$. Thus, by substituting $u = 2x^2 + 1$ and $dx = \frac{1}{4x}du$, we get

$$\int xe^{2x^2+1}dx = \int xe^u dx = \int xe^u \frac{1}{4x}du = \int \frac{1}{4}e^u du = \frac{1}{4}e^u + C = \frac{1}{4}e^{2x^2+1} + C.$$

Problem 3. For these problems, fill in the blanks and compute the integrals.

1.

$$\int x^2 e^{x^3-2}dx \implies \begin{aligned} u &= x^3-2 \\ \tfrac{du}{dx} &= 3x^2 \\ dx &= \frac{1}{3x^2}du \end{aligned}$$

$$\implies \int x^2 e^{x^3-2}dx = \int x^2 e^u \frac{1}{3x^2} du = \frac{1}{3}\int e^u du = \frac{1}{3}e^u + C = \frac{1}{3}e^{x^3-2} + C$$

2.

$$\int (3x^2-2x)(x^3-x^2)^8 dx \implies \begin{aligned} u &= x^3-x^2 \\ \tfrac{du}{dx} &= 3x^2-2x \\ dx &= \frac{1}{3x^2-2x}du \end{aligned}$$

$$\implies \int (3x^2-2x)(x^3-x^2)^8 dx = \int (3x^2-2x)u^8 \frac{du}{3x^2-2x} = \int u^8 du$$

$$= \frac{1}{9}u^9 + C = \frac{1}{9}(x^3-x^2)^9 + C$$

3.

$$\int \frac{3x^2}{x^3-1}dx \implies \begin{aligned} u &= x^3-1 \\ \tfrac{du}{dx} &= 3x^2 \\ dx &= \frac{du}{3x^2} \end{aligned}$$

$$\implies \int \frac{3x^2}{x^3-1}dx = \int \frac{3x^2}{u}\frac{du}{3x^2} = \int \frac{1}{u}du = \ln|u| + C$$

$$= \ln|x^3-1| + C$$